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Methodology for predicting vibrations induced by underground railways in 2D numerical models: a real case in Madrid, Spain

Aplicación de una metodología para la predicción de vibraciones ferroviarias en túneles en modelos numéricos 2D: un caso real en Madrid (España)

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Abstract

This paper presents a methodology for predicting vibrations produced by underground railways through 2D FEM numerical models formulated in the time domain. The numerical approach is based on the application of train loads as dynamic displacements on the rail pad and the application of load correction coefficients in 2D numerical models. This load reduction coefficient is calculated by comparing 2D and 3D static numerical models, assuming that vertical displacements are equal in both cases. This methodology has been applied in a real case in Madrid, Spain, where a reasonable fit between numerical and experimental results was found. Therefore, it could be a useful tool in early stages of design and for studying mitigation measures.

Keywords: *Railway vibrations, tunnels, building, 2D numerical models, time domain*

Resumen

En este artículo es presentada una metodología para predecir vibraciones provocadas por el tráfico ferroviario en túneles mediante modelos numéricos MEF 2D formulados en el dominio del tiempo. El enfoque numérico está basado en la aplicación de la carga ferroviaria como un desplazamiento dinámico impuesto sobre el railpad y en la aplicación de un coeficiente corrector de carga que debe ser aplicado en modelos numéricos 2D. Este coeficiente corrector es calculado mediante la comparación de modelos numéricos estáticos 3D y 2D, asumiendo que los desplazamientos verticales deben ser iguales en ambos. Esta metodología ha sido aplicada en un caso real en Madrid (España), encontrándose una razonable aproximación de los resultados numéricos con las medidas experimentales, estableciendo la misma como una herramienta útil para estudios preliminares y para el estudio de medidas mitigadoras.

Palabras clave: Vibraciones ferroviarias, túneles, edificios, modelos numéricos 2D, dominio del tiempo

1. Introduction

Recently, problems associated to vibrations generated by underground railway traffic have become increasingly relevant. In fact, the impact of vibrations on people's work and daily life is considered one of the main environmental problems of advanced societies (INCEJ, 2001; Smith et al., 2013). In this scenario, and given the complexity of the problem, the scientific community has made considerable efforts in developing numerical models. Herein, the dynamic interaction among several domains (train, tunnel, ground and building) plays an important role.

Several prediction models have been proposed, from scope models (Hansen, 2005) or analytical models (Hussein and Hunt, 2007; Hussein and Hunt, 2009 to complex numerical models, as those proposed by Clouteau et al. (2005), Gupta et al. (2007, 2009) and Hung and Yang 2010 among others.

Among the developed numerical models, those based on the finite element–boundary element coupling method (FEM–BEM) are worth highlighting. These are used for modeling the tunnel and the railway platform, while the boundary elements are used to simulate the ground. The latter offers an important advantage, because the boundary element method does not need to truncate the domain; therefore, boundaries do not need a special treatment, as in the finite element method. However, viscous boundary layers or even perfectly matched layers (PMLs) are boundary conditions widely used and compared.

In relation to the solving of equations applied to the numerical models mentioned above, there are two possibilities: the time/space domain and the frequency/wavenumber domain, where the latter is most used due to its lower computational cost.

On the other hand, the propagation of waves in the ground is a tridimensional phenomenon, so, a priori, it cannot be rigorously studied in 2D models (Metrikine and Vrouwenvelder, 2000). However, 3D models for studying underground railway vibrations have a high computational cost (Andersen and Jones, 2006). In order to deal with this inconvenient, the so-called 2.5D models have been proposed, which are expressed in the frequency/wavenumber domain through finite element and boundary element

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coupling methods, where the Fourier transform is applied in the longitudinal direction of the railway, which is assumed to be homogenous in that direction (Galvin et al., 2010; Rieckh et al., 2012). This type of models has also been developed within a unique approach of the finite element method, which means a more versatile numerical model and less complex than the former ones (Lopes et al., 2014).

The authors believe that the above models provide an adequate solution for this kind of problems, but lack the practical usefulness for technicians outside of the academic sphere, given the mathematical complexity of their expressions. Therefore, this paper proposes a simple methodology based on the Plaxis finite element software, which was applied to a real case in the RENFE commuter rail line C-7 in Madrid, Spain. The experimental measurement campaign was carried out by CEDEX (15), as a consequence of the complaints of building residents, and can be consulted in CEDEX (2003 as a consequence of the complaints of building residents, and can be consulted in CEDEX (2003) and Fernández (2014).

2. Numerical approach

2.1 Train load model

The force transmitted by each train wheel to the track is a mobile and dynamic point load. One of the main problems when studying railway vibrations in 2D models is how to represent that force, because you can only model linear and distributed loads. This major issue is easily demonstrated with the solutions defined by Boussinesq for point and linear loads on a homogenous, elastic and isotropic half-space (Das, 2008).

The wheel load is usually represented as a constant value passing over the rail (quasi-static load). Due to the strong stiffness of the rail, each rail pad receives part of this load, which generally oscillates between 25-40% of each wheel's static weight. In order to represent a wheel load in 2D models, the load received by the rail pads should be applied on them, which can be deduced by theoretical models such as: the Unold-Dischinger model (Melis, 2008), the equations of Zimmermann-Timoshenko (Melis, 2008) or the expression proposed by Krylov (1995a, 1995b). This load distribution can also be calculated with 3D static numerical models, with very similar results as those provided by the former models.

Consequently, the proposed methodology is the following: a 3D static numerical model allows calculating the loads transmitted to each rail pad, as well as the load that the most loaded rail pad receives when the wheel is located exactly on top of it. The latter is applied to a 2D static numerical model, in order to analyze the ratio between vertical displacements in both models. Assuming that they should be the same in both models, the load to be applied to 2D models is reduced, thus finding a load reduction factor to be considered in 2D models, which is the quotient between the displacements obtained with both models.

2.2 FEM Formulated in the Time Domain

In order to model the wave propagation on the ground, a 2D strain model is developed with the Plaxis software. For a full review of the mathematical expression, please refer to Plaxis. The basic equation of dynamic balance is written as a matrix equation, as follows:

$$M\ddot{u} + C\dot{u} + Ku = F \qquad \textbf{(1)}$$

Where M, C, K and F are the mass, damping, stiffness and load vector matrices, respectively. Damping is expressed as a function of the mass and the stiffness, according to a Rayleigh damping type, with the following formula: $C = \alpha M + \beta K$, where α and β are Rayleigh coefficients.

An implicit method of time integration is applied. One of the main issues when modeling "infinite" domains with a finite element model is the correct representation of the Sommerfeld condition. In fact, this method is adequate for treating finite domains, which requires a complete delimitation of the domain. Thus, special schemes should be adopted when treating boundary conditions, to prevent wave reflections from having an impact on them. In this case, viscous boundaries were applied, according to the proposal of Lysmer y Kuhlmeyer (1969). Based on this expression, the following ratios prevail along the boundary (Plaxis):

$$\sigma_n = -C_1 \rho V_p \dot{u}_n \qquad \textbf{(2)}$$

$$\tau = -C_2 \rho V_s \dot{u}_t \qquad \textbf{(3)}$$

Where ρ is the materials' density and V_p and V_s is the speed of waves P and S, respectively. C_1 and C_2 are dimensionless relaxation coefficients that are introduced to improve the absorption effect. Analytical studies carried out by Lysmer y Kuhlemeyer (1969) estimate that the use of C_1= C_2=1 entails a perfect absorption when orthogonal waves reach the boundary. Thus, these values are valid for 1D problems, but in 2D and 3D cases, the absorption depends on the wave's angle of incidence. Therefore, Plaxis recommends using C_1=1 and C_2=0.25, which are the values considered in this paper. Furthermore, Cohen (1980) has demonstrated that this kind of boundaries are not very sensitive to the values of C_1 and C_2.

3. Case study

3.1 Overall description

The case study corresponds to an old surface tunnel (1860), located in Madrid, Spain, belonging to the RENFE commuting rail network. Figure 1 shows the localization of the tunnel and Figure 2 indicates the cross-section and floor plan of the problem.

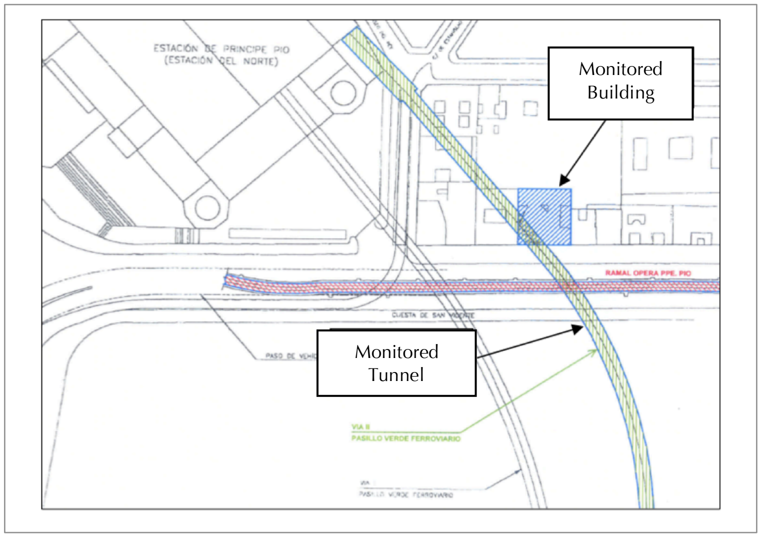

Figure 1. *Localization of the case study*

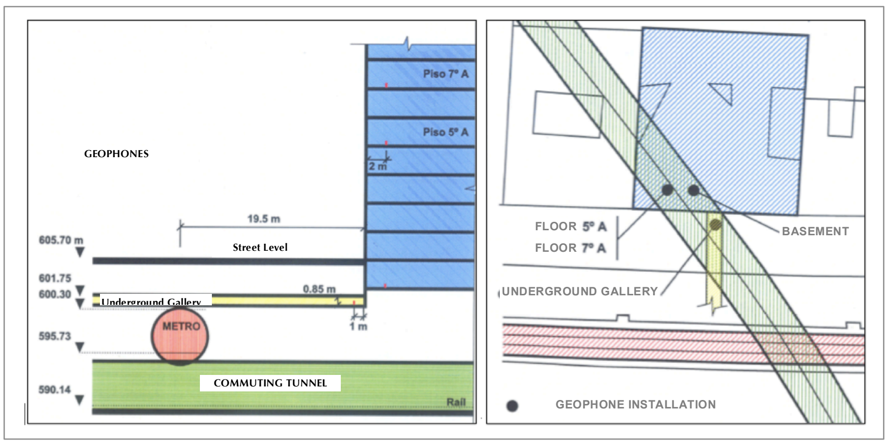

Figure 2. *Drawing of the tunnel-building system (cross-section; floor plan)*

The tunnel lining is made of masonry, a common typology in old tunnels. A Young's Modulus of 5 GPa and a Poisson coefficient of 0.2 were considered. Although the values of 5 GPa could be somewhat higher for a masonry of the 19th siècle, a sensitivity analysis was made for this value, and no relevant differences were appreciated in the numerical results. Figure 3 shows the tunnel geometry.

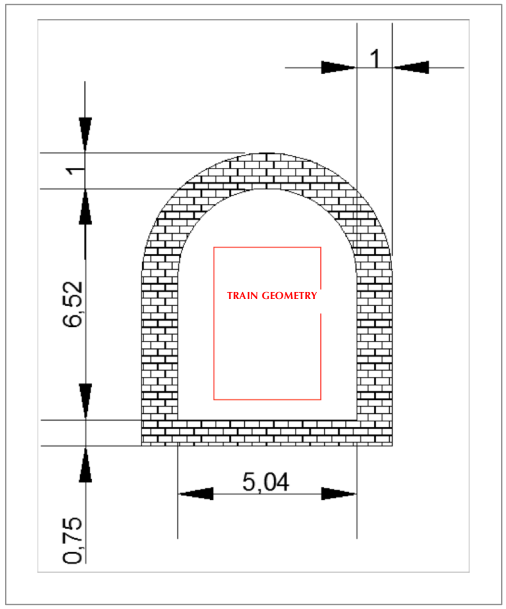

Figure 3. *Tunnel Geometry*

The railway platform is a STEDEF-type of track. The rail is UIC-54, with a Nabla fastening system and the separation between sleepers is 0.6 m. Figure 4 shows a detail thereof. The elastic dynamic properties of the railway elements are summarized in Table 1.

The building was built in the 1950's. Figure 5 shows the façade, floor plan and section view. It has been assumed that the building is founded on a slab. Table 2 shows the building's elastic properties

The properties of the ground were deduced according to the studies carried out by Melis (2008) concerning the geotechnical characterization of the soil for recent tunnel constructions in the case study surroundings. A reasoned discussion of these parameters can be consulted in Fernández (2014). The geotechnical profile of the ground is formed by two soils: sand (the first 10 meters) and clay (considered indefinite in depth). Table 3 shows their geotechnical parameters. It is well known that, in general, vibrations transmitted by underground railway traffic imply very small strains in the ground; therefore, a model with elastic and linear behavior would be accurate enough. Nevertheless, this case considered a Mohr-Coulomb model, because the tunnel and building constructions were numerically simulated, so that it can take into account the potential plastic zones in the soil. The use of advanced constitutive models of soil behavior (such as Hardening Soil SMALL) for the numerical modeling of the tunnel and building constructions, does not imply a relevant difference in relation to more simple models like the Mohr-Coulomb, provided that the soil strain modulus is considered in very small strains (E_0), as shown by Fernández y Medina (2015).

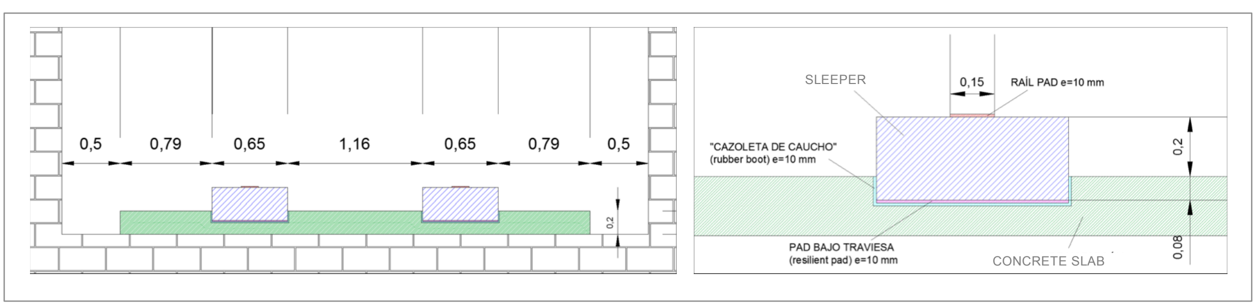

Figure 4. *Detalle esquemático de la plataforma ferroviaria*

Table 1. *Track Properties*

	RAIL UIC-54	PAD	SLEEPER	RESILIENT PAD	RUBBER BOOT	SLAB
Type of Behavior	Linear-Elastic	Linear-Elastic	Linear-Elastic	Linear-Elastic	Linear-Elastic	Linear-Elastic
Dimensions (m)	0.14*0.12* "indefinite"	0.14*0.12* 0.01	0.65*0.2* 0.28	0.65*0.2* 0.01	"variable"	-
Thickness (m)	-	-	-	-	-	0.2
γ (kN/m^3)	32.40	9.5	25	9.5	9.5	25
E (kN/m^2)	$210*10^6$	$71.43*10^3$ (K_{din}=200 kN/mm)	$30*10^6$	$3.07*10^3$ (K_{din}=40 kN/mm)	$22.22*10^3$ (K_{din}=40 kN/mm)	$30*10^6$
ν	0.3	0	0.2	0	0	0.2

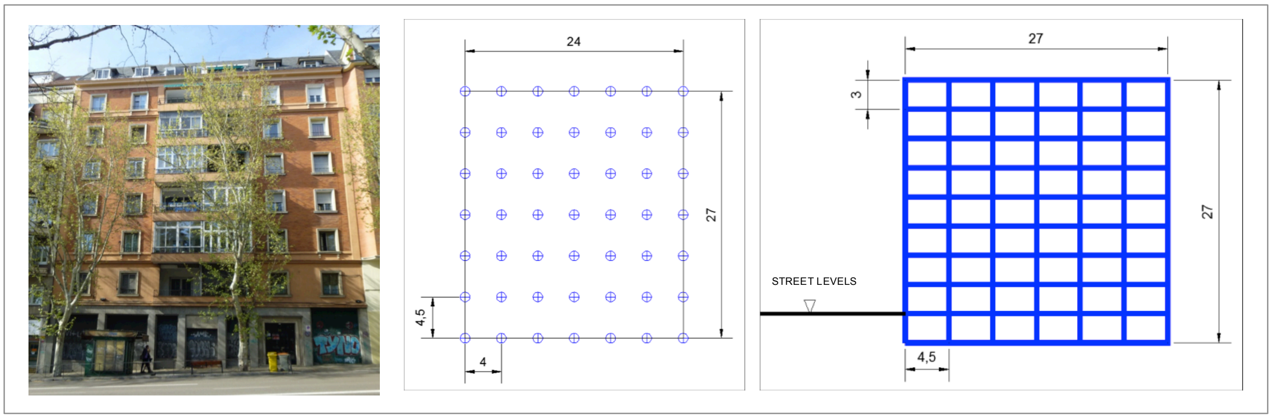

Figure 5. *Monitored building (image of the facade; structural floor plan (in m); structural section (in m))*

Tabla 2. *Building Properties*

	FOUNDATIONS AND FLOOR SLABS	PILLARS	BASEMENT WALLS
Structural Type	Plate	Beam	Plate
Behavior Type	Elastic-Linear-Isotropic	Elastic-Linear	Elastic-Linear-Isotropic
γ (kN/m^3)	25	25	25
Thickness (m)	0.3	-	0.4
Dimensions (m)	-	0.35*0.35	-
E (kN/m^2)	$30*10^6$	$30*10^6$	$30*10^6$
ν	0.2	-	0.2
G (kN/m^2)	$12.5*10^6$	-	$12.5*10^6$
I (m^4)	-	$1.251*10^{-3}$	

Table 3. *Geotechnical Parameters of the Soil*

	SAND	CLAY
Behavior Model	Mohr-Coulomb	Mohr-Coulomb
γ_{ap} (kN/m^3)	20	21
E (kN/m^2)	$320*10^3$+6000z	$550*10^3$+8000*(z-10)
ν	0.3	0.3
φ'	35°	30°
c' (kN/m^2)	5	30

The experimental measurements were made by CEDEX during the passing of the RENFE train type 446 (with double configuration). Figure 6 shows the geometry and loads by axle.

For a more detailed report regarding this case, please refer to the information in CEDEX (2003) and Fernández (2014).

3.2 Experimental measurement campaign

The experimental measurement campaign consisted in measuring the running vibration level at different circulation speeds. The vertical of the rail and the tunnel slab were measured, as well as the speed in the 5^{th} and 7^{th} floor of the building. Measurements were made by CEDEX in 2002. For a more detailed information, please refer to CEDEX (2003) and Fernández (2014).

For the case herein, the train speed was 14.25 m/s.

4. Numerical model

The numerical models used, both in 2D and 3D, were developed with the Plaxis software and they are represented in Figures 7 and 8, respectively. Finite elements are triangular (2D) and tetrahedral (3D), with second-order interpolation function in both cases.

Regarding the size of finite elements, the criterion of six boundary elements per wavelength was used, according to Unterberger y Hochgatter (1997). The maximum considered frequency was 80 Hz.

Three finite elements were used for modeling the tunnel lining, in accordance with Unterberger y Hochgatter (1997) and Clouteau et al. (2005).

In relation to the time step, it was determined by the maximum speed of the P wave in each element (Unterberger and Hochgatter, 1997), thereby resulting in a dynamic time step of $2.5 \cdot 10^{-5}$ s, imposed by the sleeper elements.

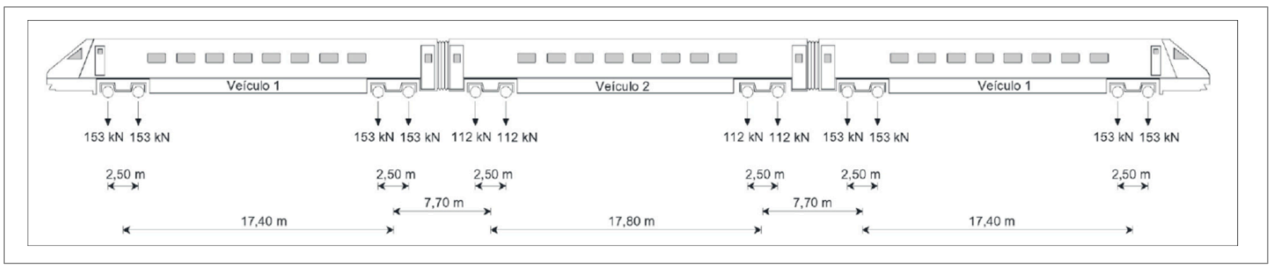

Figure 6. *RENFE Train 446 – Simple configuration*

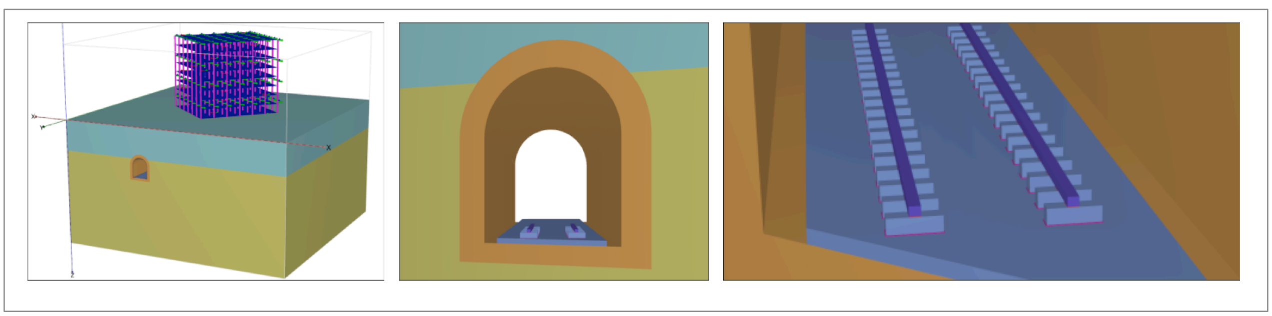

Figure 7. *3D Numerical Model (3D overview; tunnel; railway platform)*

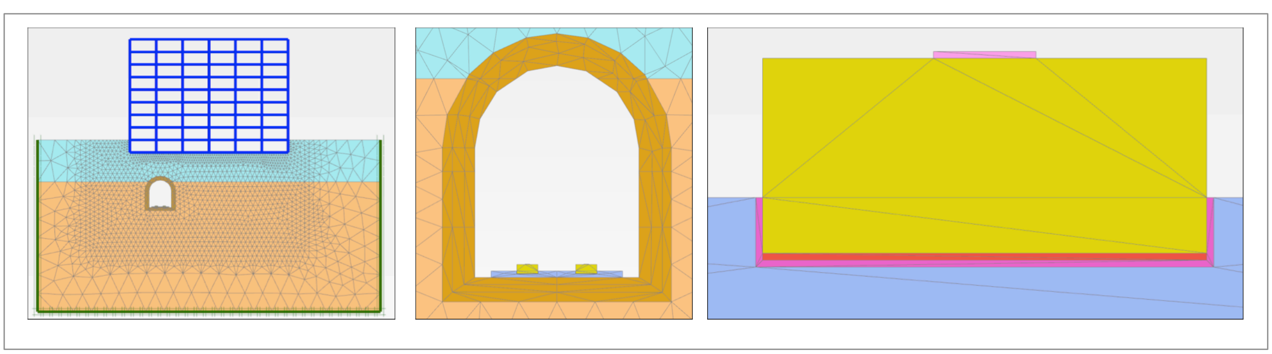

Figure 8. *2D Numerical Model (global model; tunnel; sleeper)*

4.1 Static models and calculation of the reduction coefficient

For calculating the load correction coefficient, the 3D numerical model was compared with the 2D model, both of them static. In the first, two point loads of 7.65 tn were applied, one above each rail exactly on top of a rail pad. Figure 9 shows the load supported by each rail pad.

The load applied to the most loaded rail pad is 1.994 tn (26% of the wheel's static load), which is the load applied in the 2D model. Table 4 shows compared results of inputs from 2D and 3D models.

According to the table above, the reduction coefficient is 0.35 for the slab and 0.112 for the building. This means that, in order to find an input in the 2D model that is equal to that obtained in the 3D model, the load applied in the 2D model should be reduced by a factor of 0.35 for the slab and 0.112 for the building. In this way, two dynamic numerical models have been performed, one for the slab that reduces the load by a factor of 0.35 and another one for the building, reducing the load by a factor of 0.112.

4.2 Dynamic models

Given the fact that actual rail acceleration measurements are available and that the Plaxis software allows entering the load as a dynamic displacement, it was decided to enter the actual displacement of the rail in the 2D dynamic model, in order to validate the methodology. This displacement was also corrected according to the calculated reduction coefficients.

The Rayleigh damping approach was used, which is shown in Table 5. A 0.5-50 Hz frequency interval was considered to take into account the relevant frequency range for this case. However, the frequency range does not have a great influence on the numerical results, merely a small difference in the frequency domain, as appreciated in Fernández (2014).

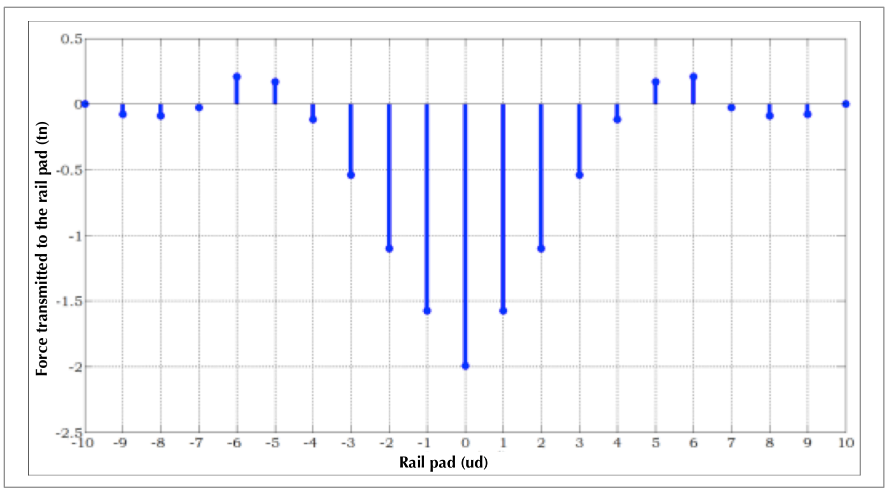

Figure 9. *Load distribution on rail pads in the 3D static numerical model*

Table 4. *Inputs (mm) in 2D and 3D Static Models*

	3D Model	***2D Model (100% of the load applied to the most loaded rail pad)***	***Reduction Coefficient***
RAIL	*0.676*	-	-
TUNNEL SLAB	*0.072*	*0.207*	*0.350*
5TH FLOOR	*0.012*	*0.107*	*0.112*
7TH FLOOR	*0.012*	*0.107*	*0.112*

Table 5. *Rayleigh damping*

ELEMENT	DAMPING (%)	FREQUENCY RANGE (Hz)	DAMPING COEFFICIENTS	
			α	β
Rail pad and resilient pad	10.0	0.5-50	0.6220	0.00063030
Sleeper and slab	2.5	0.5-50	0.1555	0.00015750
Rubber boot	5.0	0.5-50	0.3110	0.00031515
Lining	2.5	0.5-50	0.1555	0.00015750
Clay	2.0	0.5-50	0.1244	0.00012600
Sand	2.0	0.5-50	0.1244	0.00012600
Building (walls, pillars and floor slab)	2.5	0.5-50	0.1555	0.00015750

The damping mechanism in concrete elements (sleeper, tunnel slab and building) and masonry elements were considered according to Newmark and Hall (1982). On the ground, it was estimated according to Ishibashi and Zhang (1993) and in the rail pad, according to Maes et al. (2006); Thompson and Verheij (1997).

En la Figura 10 se muestran las dimensiones y el modelo dinámico 2D de elementos finitos empleado para las simulaciones numéricas.

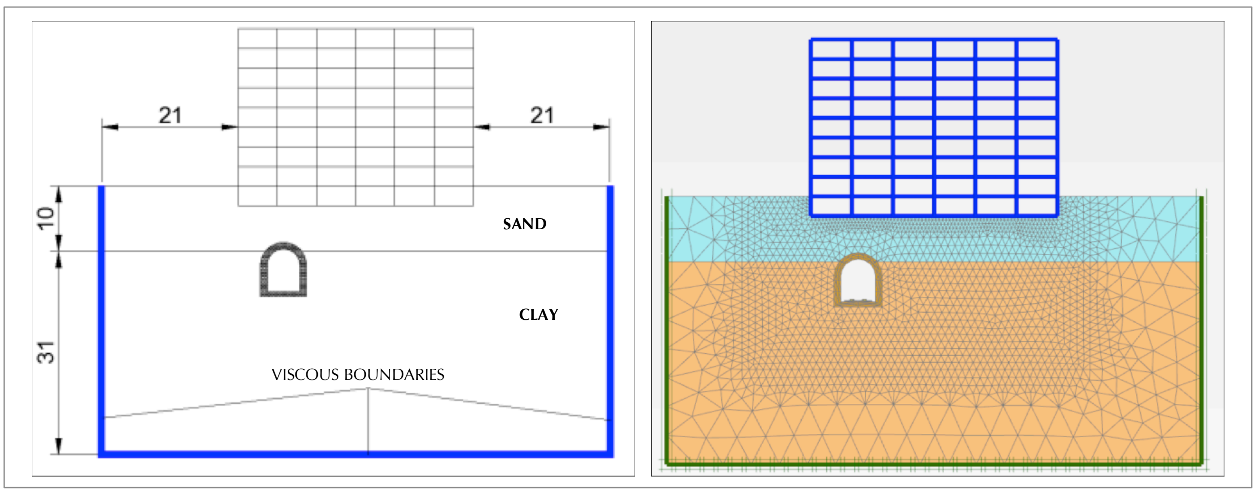

Figure 10. *2D Dynamic model for numerical simulations (geometric diagram (distance in meters); 2D numerical model)*

5. Results and discussion

Figure 11 shows a comparison between numerical and experimental results in the tunnel slab, where a reasonable fit was found. However, the numerical model overestimates the response in the frequency interval between 45 and 55 Hz. This is also evident in the time domain, although from a general point of view it is possible to conclude that differences are acceptable. The latter may be due to possible uncertainties in the stress-strain and damping parameters of the rail pad and the elastomeric pad under the sleeper or to the own limitations of the numerical models. Additionally, since a 2D model expressed in plane strain was used, it does not consider the effect of geometrical damping, which could also explain the differences.

Figure 12 shows a comparison between numerical and experimental results in the time domain, in the 5th and 7th floor of the building. Logically, these differences are more relevant than in the tunnel slab, although they are still acceptable. A reasonable fit is observed in the vibration speed peaks, but greater differences are observed off these peaks.

Figure 13 shows a more evident comparison in the building regarding the frequency domain. Differences are not especially relevant, although they are clearer in the 7th floor. In both cases, the numerical model underestimates the response in practically all frequency ranges. Nevertheless, the numerical model reproduces the experimental measurement acceptably. In the experimental measurement, the speed peak around 12 Hz in the 7th floor is worthy of mention. The authors believe that there is no logical explanation for this peak value and it could be due to a noise or even a work frequency of some kind of device in the surrounding of the measurement point.

A more useful comparison is to present the vibration levels in the building in third to eighth frequencies (Figure 14), where an acceptable fit is observed, except in the frequency interval around 10 Hz. Outside of this interval, differences barely exceed 10 dB. Although this value is not insignificant, several studies have shown that it is difficult to approximate numerical models to the actual response below this value (Hunt and Hussein, 2007; Gupta et al., 2009 and Jones et al., 2012).

Figure 14 indicated that the numerical model underestimates the vibration level in practically all frequency bands. This may be due to the following reasons:

- The ground is not a perfectly homogenous medium, its stiffness and damping may vary locally; this fact is more relevant in high frequencies.
- The methodology is based on the correlation of static 2D and 3D models, which do not take into account the dynamic amplification factor.
- The Rayleigh damping depends on the frequency; therefore, in high or low frequencies, it can be too high.
- A 2D numerical model cannot accurately represent the vibration of a building, which is still a tridimensional structure.

However, from a general point of view, this methodology generates results with an acceptable fit level and its computational cost is affordable. In fact, the calculation time for 2D numerical models was 1 hour per each second of dynamic time, in a computer with a processor Intel ® Core^{TM} i7-2600K CPU @ 3.4 GHz.

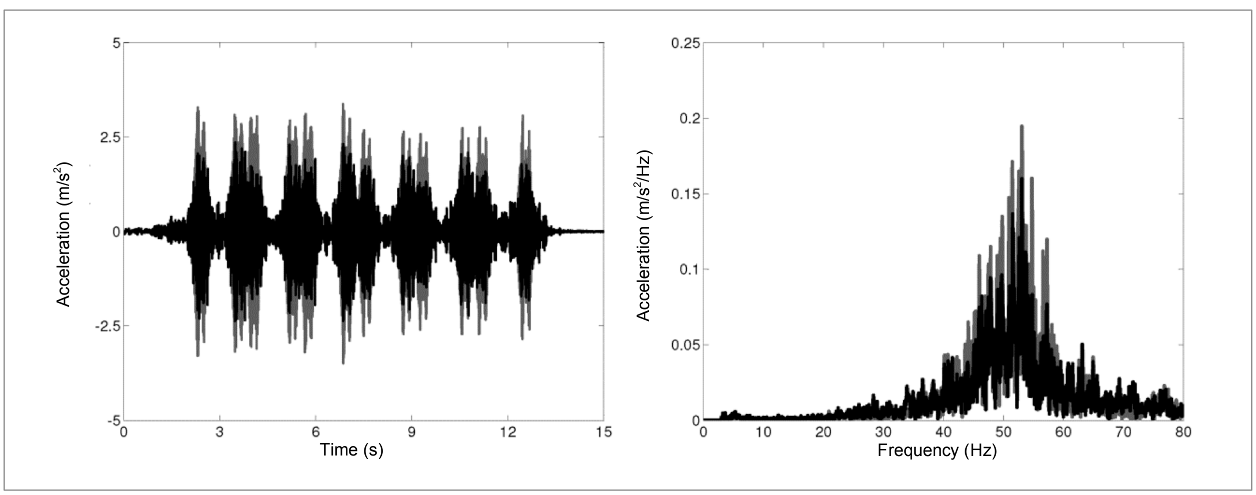

Figure 11. *Vertical acceleration in the railway platform slab (black line: experimental measurement; gray line: numerical result)*

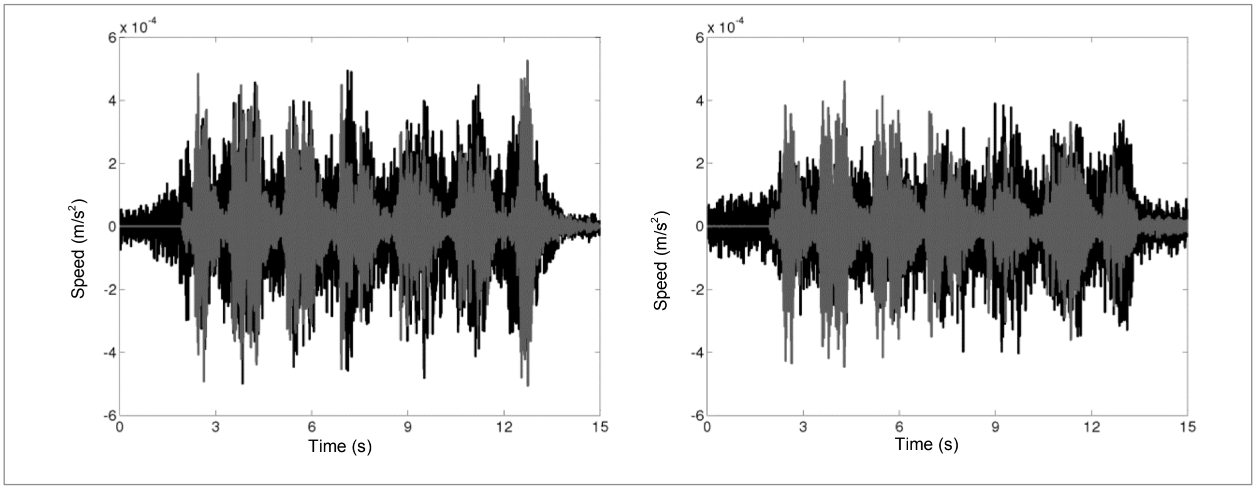

Figure 12. *Vertical speed in the building: left, 5th floor; right, 7th floor (black line: experimental measurement; gray line: numerical result)*

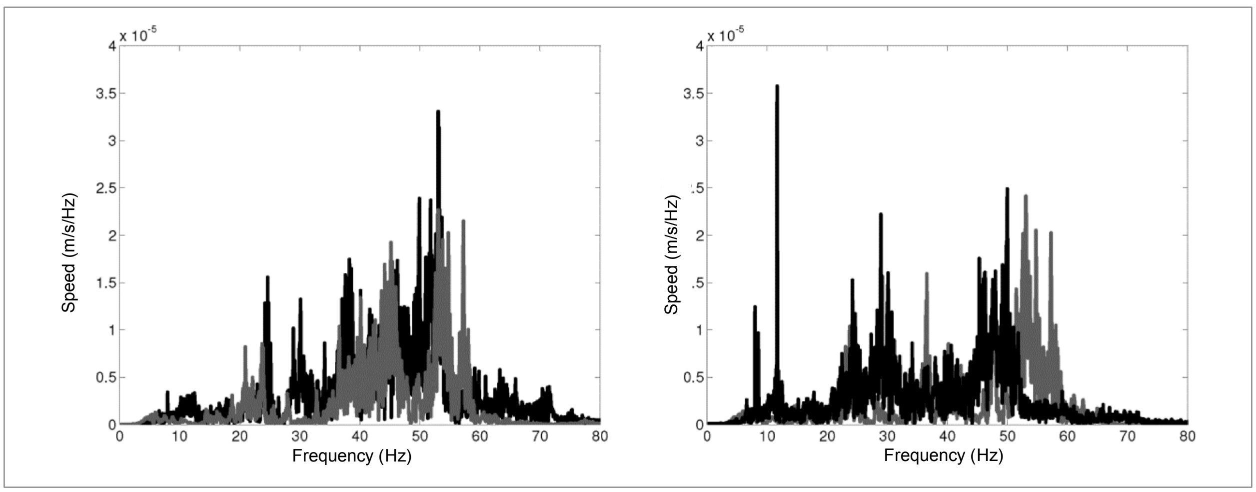

Figure 13. *Vertical speed in the building: left, 5th floor; right, 7th floor (black line: experimental measurement; gray line: numerical result)*

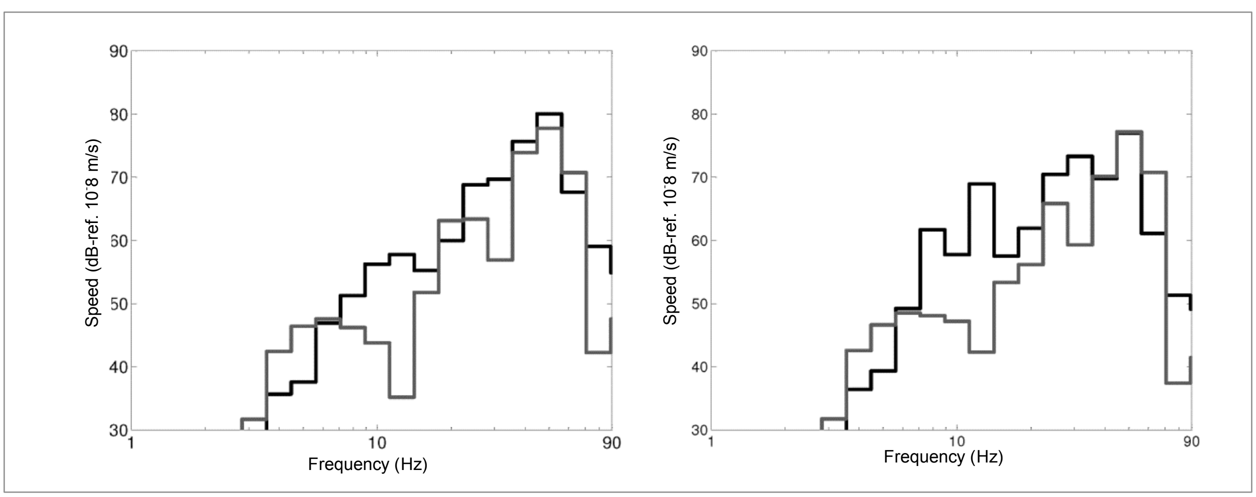

Figure 14. *Vertical speed in the building: left, 5th floor; right, 7th floor (black line: experimental measurement; gray line: numerical result)*

6. Conclusions

This paper has presented a methodology for predicting vibration levels caused by underground railway traffic through 2D finite element numerical models with the Plaxis software. This methodology has been validated in a real case located in Madrid (Spain), where numerical results show a reasonable fit with those experimentally measured, both in the interior of the tunnel and in the building.

The applied numerical approach can be considered a useful prediction tool with an easy implementation by technicians or professionals that usually design and project underground railway lines in urban areas and who are not used to handling complex numerical models (for example, FEM-BEM coupling models), which are limited to the academic sphere. It is noteworthy that, in addition to the quasi-static load, the study took into account the total dynamic load transmitted by the train (due to defects in the rail and wheels, joints in rails, among others), which has entailed a reasonable fit of the numerical response with the values actually measured. The goodness of fit of the numerical results will greatly depend on the knowledge about the factors generating the dynamic load.

Obviously, the methodology has some limitations, since it is based on the correlation of static numerical models, which do not consider the dynamic amplification factor. This factor could be relevant in those cases where natural vibration frequencies of the building or the tunnel are close to the main excitation frequencies of the railway load. Under these circumstances, the differences might be slightly higher than those shown in the present paper.

Nevertheless, the proposed methodology can be considered an adequate tool for a preliminary study of vibration levels caused by railway traffic, with an affordable computational cost, which is a limiting factor for many railway dynamic analyses. Furthermore, it allows studying potential mitigating measures in a fast and effective way

7. Acknowledgements

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