

Influence of web stiffeners in the bearing capacity of steel castellated beams subjected to failure by web post instability

Influencia de los rigidizadores de alma en la capacidad última de vigas alveoladas de acero sujetas a la ruina por inestabilidad del montante de alma

W. Vieira ^{1*}, G. Verissimo **, J. L. Rangel **, J. Lopes **

* Universidade Federal de Itajubá, Minas Gerais. BRAZIL

** Universidade Federal de Viçosa, Minas Gerais. BRAZIL

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Abstract

This work presents a numeric study using the Finite Element Method for castellated beams manufactured from Brazilian steel rolled I-sections, which collapse by web post buckling. After the beam analysis for the limit buckling state, other analyses were carried out, adding web post stiffeners in the region that would suffer the buckling. The results show significant influence of the stiffeners in the bearing capacity and in the collapsing mode presented in the analyzed beams.

Keywords: Castellated beams, finite element methods, web post buckling, web stiffeners

Resumen

Este trabajo presenta un estudio numérico por el método de los elementos finitos para vigas alveoladas fabricadas a partir de perfiles laminados en Brasil, que colapsan por inestabilidad del montante de alma. Después de revisar las vigas que han sufrido el colapso por inestabilidad del montante de alma, otros análisis se llevaron a cabo, añadiendo rigidizadores en la región que sufrirá el pandeo. Los resultados muestran la influencia significativa de los rigidizadores en la capacidad de soporte y en el modo de colapso presentado en las vigas analizadas.

Palabras clave: Vigas alveoladas, método de los elementos finitos, pandeo del montante de alma, rigidizadores del alma

1. Introduction

Steel castellated beams emerged in the first half of the 20th century in Europe, due to the necessity of beams with higher depth than the hot rolled I-sections manufactured by the industry at that time. According to Grünbauer (2014), in the beginning of the '30s, castellated beams with a free span of 12 meters were manufactured by Skoda Company in Czech Republic. Since the beams are welded, their development is directly connected with the creation of welding technology at the end of the '20s .

Conventionally, castellated beams are manufactured by cutting a hot-rolled steel beam in zig-zag, splitting it lengthwise in a rack-shaped pattern into two halves which are then shifted of a specific distance and then welded together at the tops of the teeth. The result is a beam deeper than the

original rolled I-section it is made from, with a sequence of hexagonal holes in the web, with the same steel weight (Figure 1a). It is possible to obtain an even deeper castellated beam welding square or rectangular steel plates between the halves, resulting in a sequence of octagonal holes (Figure 1b).

A few decades later, castellated beams stopped being used due to the increasing cost of labor in developed countries, since the manufacturing process was fully manual.

Recently there has been a resurgence of interest for steel castellated beams. Due to the technological development in the automation of manufacturing processes, these elements are regaining competitiveness in the market (Figure 2).

¹ Corresponding author:

Eng. Civil (UFV, 2009), Doutor em Eng. Civil (UFV, 2015), Prof. Adjunto (UNIFEI-Campus Itabira).

E-mail: vieira@unifei.edu.br



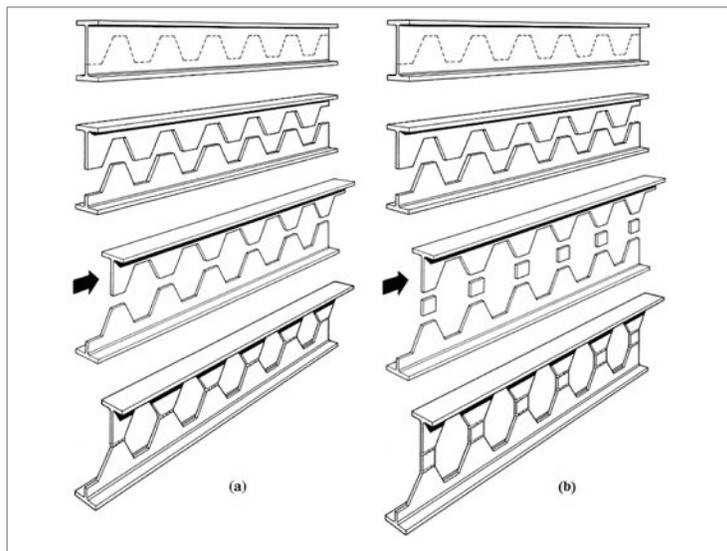


Figura 1. Manufacturing scheme for castellated beams. (a) Without intermediate plate and (b) with intermediate plate (Grünbauer, 2014)

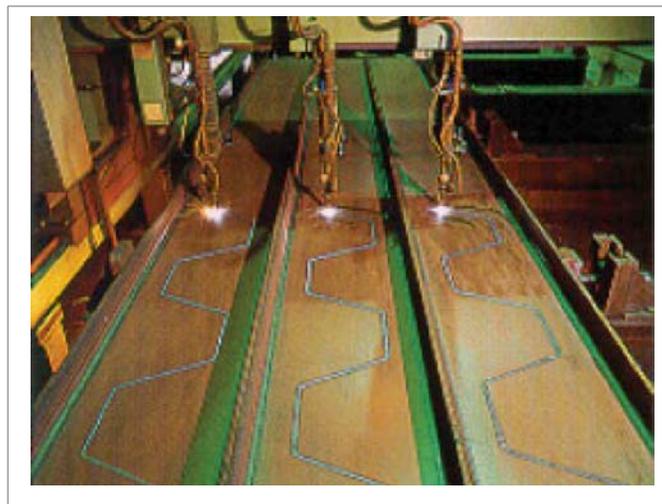


Figure 2. Castellation beam manufacturing: CNC cut (Gemperle, 2007)

Many different theoretical and experimental researches have been conducted in the past century aiming to explain the structural behavior of castellated beams. However, although these studies had provided procedures to design and calculate some specific castellated beams from the rolled I-beams manufactured at that time, they were carried out in bases today considered outdated, especially from elastic analysis and made compatible with the Allowable Stress Design method. Moreover, over a period of about 30 years when castellated beams remained forgotten, the steels and the rolled sections produced by the industry have been modified so that the structural behavior of current profiles may present differences in relation to older ones.

In a relatively recent experimental study, Zaarour and Reedwood (1996) conducted bending tests with castellated beams produced from Bantam beams, a slender rolled beam series manufactured by Chaparral Steel in the United States. These beams had sheets with slenderness higher than the older rolled I-sections and displayed failure modes related to instability in the tests, unlike the oldest rolled profiles, which had a more compact section.

With the relatively recent activation of the profile rolling mill by Gerdau Group, in 2002, some of the new rolled I-sections produced in Brazil also have slender sheets, with web slenderness beyond the range covered by experimental studies conducted in the past, as shown in Figure 3.

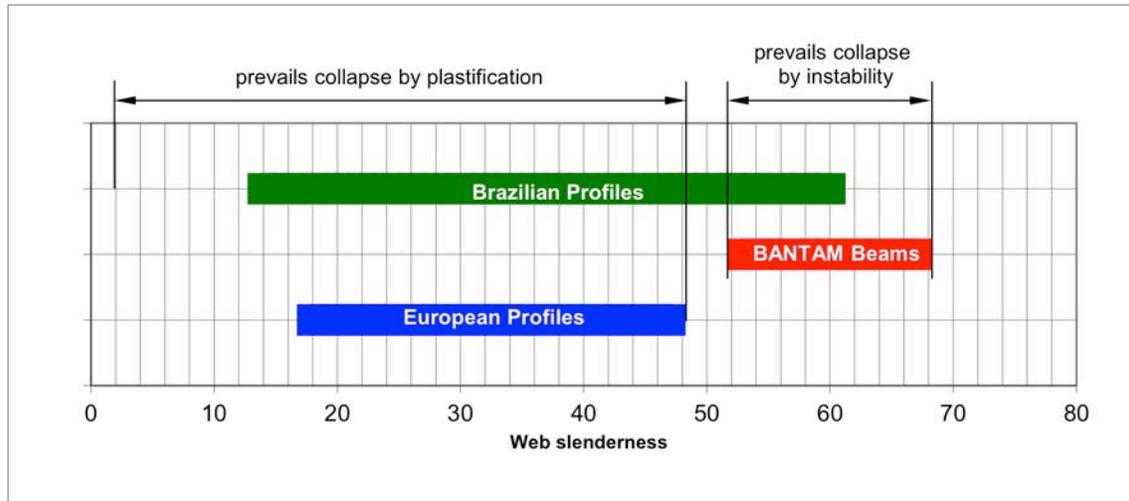


Figure 3. Web slenderness range for European, Bantam and Brazilian steel rolled beams

This particularity drew the attention of Brazilian researchers who have been developing new studies on the structural behavior of castellated and cellular beams (beams with circular openings).

Abreu et al. (2010) presented a study related to the lateral-torsional buckling in cellular beams. Bezerra et al. (2010) presented a similar study for castellated beams. Silveira et al. (2011) performed a numerical study for castellated and cellular beams with emphasis on the failure modes for which the steel reaches yielding and presented a formulation for checking the bearing capacity of the beams for the various ultimate limit states applicable. Vieira et al. (2011) developed a numeric model to simulate the structural behavior of castellated beams. They subsequently conducted a study focusing on the collapse by instability of the web posts, noting that some features related to geometry and loading of the beams reduce the buckling load of the web post.

This paper presents a study of the influence of web stiffeners in the structural behavior of castellated beams, through a finite elements numerical model developed with the ABAQUS software. For analyzed models, the structural response for various settings of web stiffeners was studied, in order to evaluate the performance of the stiffeners to prevent out of plane buckling of the web post and increase bearing capacity of the beams.

2. Definitions and symbology

The constituent parts of a generic castellated beam with intermediate plates and the symbology used in this work are displayed in Figure 4. The openings, or holes, are often referred to as alveoli and the beams as alveolar beams.

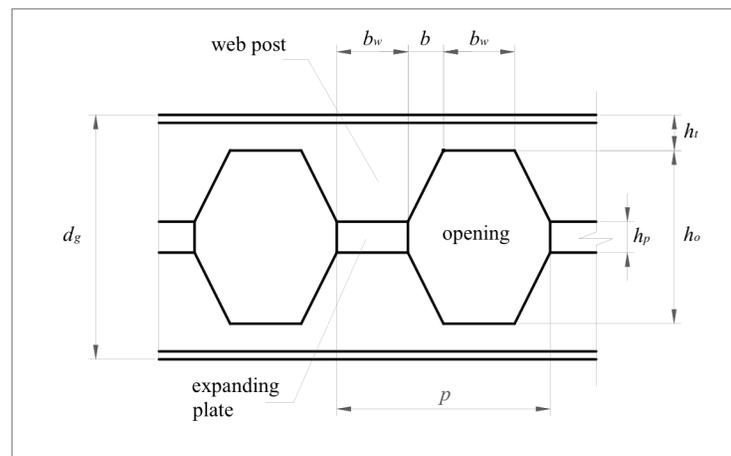


Figure 4. Symbology related to dimension and spacing of openings in castellated beams



Figure 5 presents the geometric characteristics for the three patterns of castellated beams which are the most used ones: Anglo-Saxon, Peiner and Litzka.

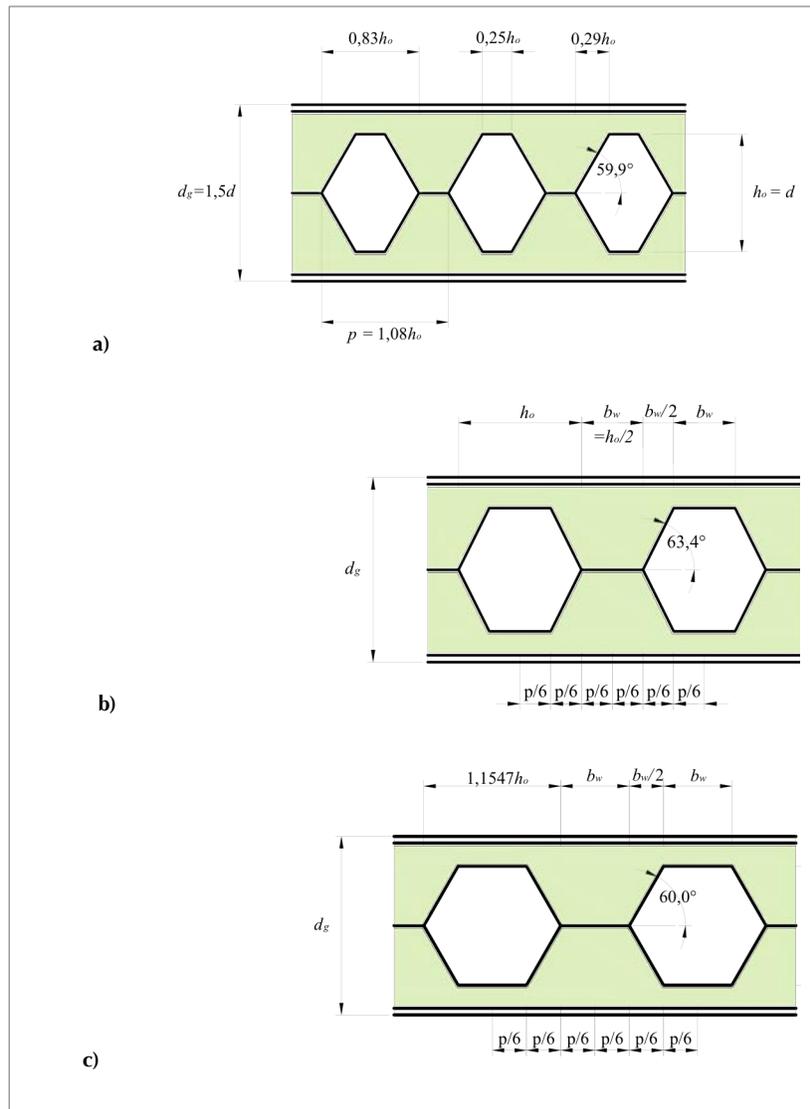


Figure 5. Geometric patterns for castellated beams: (a) anglo-saxon; (b) Peiner; (c) Litzka

3. Failure modes in castelated beam

The presence of openings in castellated beams causes a change in their structural behavior, leading them to present failure modes different from those observed in solid-section beams.

Experimental studies carried out in the past decades identified some different failure modes in castellated beams (Kerdal e Nethercot, 1984; Demirdjian, 1999). The most frequent failure modes in castellated beams are: formation of a flexure mechanism (hinge), formation of Vierendeel mechanism, rupture of a welded joint in a web post, lateral-torsional buckling of an entire span and web post buckling.

- Flexure mechanism – in castellated beams with compact sections, the maximum load is obtained by yielding of the tee-sections above and below the alveoli;
- Vierendeel Mechanism – failure mode characterized by the formation of four plastic hinges in the openings corners with the distortion of the panels, by the combination of normal and shear stresses. It happens more often in beams with small spans, with long welding length and tee-sections with lower depth in relation to the expanded depth of the beam. (Kerdal e Nethercot, 1984);
- Rupture of a welded joint in a web post – it usually occurs when the shear stress in welding regions reaches the material yield stress;
- Lateral-torsional buckling – castellated beams with an unbraced length insufficient to contain the compressed tee-section can fail by lateral-torsional buckling.

- Web post buckling – presented in the section 0.

3.1 Web post buckling

Kerdal and Nethercot (1984) describe various cases of castellated beams failing due to web post buckling (WPB), where most happened in the inelastic range.

Delesques (1968) investigated the WPB assuming elastic behavior, but concluded that the elastic buckling rarely happens. Aglan and Redwood (1974) analyzed castellated beams using a finite difference approximation and assuming a elastoplastic material with hardening. They found out that the web posts reached an advanced state of yielding before the occurrence of the WPB.

The instability of the web post can happen in two ways: by shearing of the minimum section or by web compression.

3.1.1 Web post buckling by shearing

By analogy with a Vierendeel beam, the web post of an alveolar beam has internal forces as shown in Figure 6. The shearing force F along the welded joint causes flexure of the web post. The AB edge of the web post is subjected to tensile stresses, while the edge CD is subjected to compressive stresses, causing the WPB. When it occurs, the lateral displacement of some portions of the web will be followed by the torsion of the diagonal line xx' (Figure 6). This failure mode is denominated web post buckling by shearing.

Figure 7 displays the state of a cellular beam tested by Nadjai et al. (2008) that failed by instability in web post caused by shear.

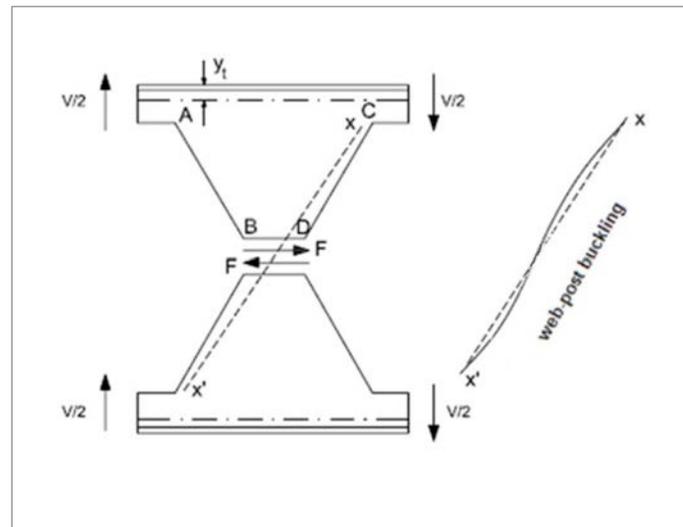


Figure 6. Web post buckling by shearing (adapted from Kerdal and Nethercot, 1984)





Figure 7. Web post buckling of a composite cellular beam (Nadjai et al., 2008)

The web post buckling by shearing is influenced by the following geometric relations: h_o/d_g ; h_o/b_w ; b_w/t_w ; h_p/h_o ; h_p/t_w (Zaarour and Redwood, 1996; Redwood and Demirdjian, 1998), where: h_o is the alveoli height; d_g is the castellated beam depth; h_p is the expanding plate height; b_w is the minimum width of the web post and t_w is the web thickness.

3.1.2 Web post buckling by compression

This failure mode usually happens in points where a concentrated load is applied over the web post or in the region of the supports, especially when there are no stiffeners. The WPB by compression is similar to the web corrugation that can be observed in solid-section beams subjected to a concentrated load. Unlike what happens in the web post buckling by shearing, the lateral displacement of the web post that buckles by compression is not accompanied by torsion. Figure 8 shows the deformed configuration obtained through finite elements models of castellated beams that suffered web post buckling by compression.

When a castellated beam is subjected to a uniformly distributed load, the web posts are subjected to horizontal and vertical forces. The horizontal forces happen due to the shear force that will produce shearing, flexure or web post buckling by shearing. The vertical forces in the web post are a result of the direct action of the load applied on it that can cause buckling by compression, similarly to what happens in a column.

In the web posts at the ends of the beam, horizontal forces usually are preponderant in relation to vertical forces, while in the web posts near to the span center - where the shear is null - the compression forces tend to rule the behavior. The compression forces in the web posts become even more significant in case of short length beams, when the distributed load that the beam supports is higher and consequently the compression load is bigger.

4. Numeric model

A numeric model was developed in the software ABAQUS, using the Finite Element Method, in order to simulate the structural behavior of castellated beams, having the following characteristics (Vieira et al., 2011):

- (a) thin shell tridimensional model, representing the middle surfaces of the steel plates that compose the castellated beams;
- (b) mesh with S3 type elements in the triangular regions close to the openings and S4R elements in the rest of the beam (Figure 9);
- (c) the mesh objectivity was reached using element with 1 cm sides;
- (d) to represent the steel, a constitutive model for a perfectly elastoplastic material without hardening was adopted;
- (e) an elastic buckling analysis was carried out to obtain the eigenvalues and correspondent eigenvectors (the eigenvalues represent the load factor for which the instability occurs and the eigenvectors the buckling mode associated);
- (f) after the instability analysis, a material and geometric non-linear analysis was carried out to simulate the behavior of the structure along the loading process, including the post-peak phase. The characteristics of this analysis are listed below:
 - i. the analysis was performed using the modified Riks method, applying an initial imperfection to the model corresponding to the buckling mode selected in the elastic buckling analysis, to find the maximum resistance of the structure; the agreement with experimental results was better when an imperfection equal to $d_g/2000$ was admitted;
 - ii. a simple distribution of residual stresses was used, considering uniform tensile stress in the flanges and uniform compression stresses in the web, calculated by the parabolic distribution from Young (1972 apud SZALAI AND PAPP 2005), so that the resultant force in the web and in the flanges of the simplified model be equal to the parabolic model; the residual stresses utilized for the web and for the flanges can be calculated using the Equations 1 and 2, respectively (in MPa):

$$f_{r,alma} = \frac{100 t_w (h - t_f)}{9 t_f \cdot b_f} + \frac{230}{3} \quad (\text{compression}) \quad (1)$$

$$f_{r,mesa} = \frac{225 t_w (h - t_f)}{4 t_f \cdot b_f} - \frac{25}{3} \quad (\text{tension}) \quad (2)$$

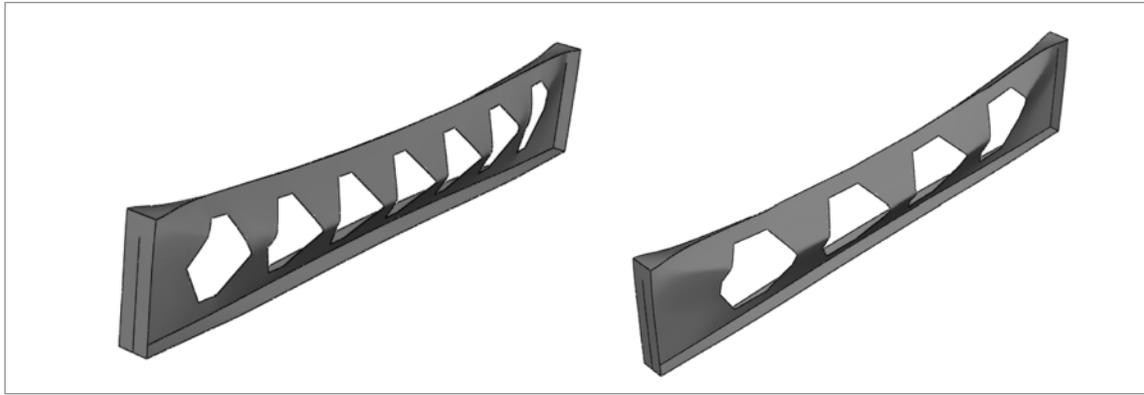


Figure 8. Aspect of castellated beams that suffered web post buckling by compression

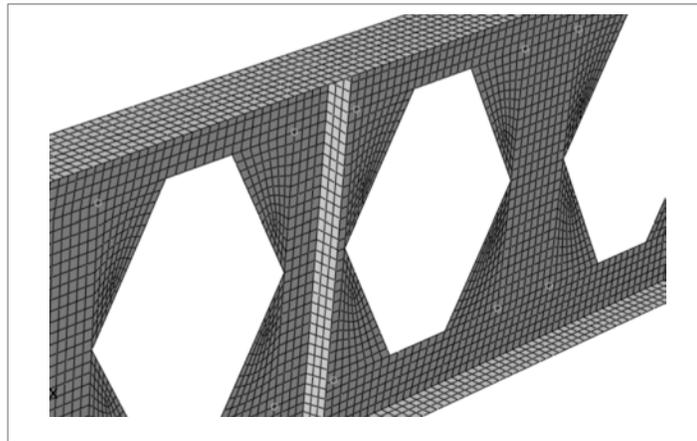


Figure 9. Finite elements mesh of a model with quadrilateral and rectangular elements

The numeric method was validated by the analysis of 14 castellated beams from four distinct experimental programs which results are presented in Vieira et al. (2011).

5. Numeric analysis of the influence of web post stiffeners

A numeric study containing 14 castellated beams was conducted, with different geometry and loading configurations and the following characteristics that were shared by all the beams:

- original rolled I-section W360x32,9 from Gerdau, with expansion rate equal to 1,5;
- simply supported beams with one of the supports restricting the longitudinal displacement;
- continuous lateral bracing along the flanges axis, to avoid lateral-torsional buckling;
- steel with $f_y = 345$ MPa;
- span of 3000 mm, in order to guarantee fail by web post buckling;
- alveoli distribution in a way such as extreme web posts had at least the width of intermediate web posts;
- web stiffeners inserted in the supports and in the points of application of concentrated load.

The other characteristics of the analyzed beams are presented in Table 1.

In the beams submitted to an uniformly distributed load (M1 a M6), the failure occurred by WPB by compression, and the largest lateral displacement has been observed in the central web posts. In these cases, a web stiffener was welded in the center of the span, as shown in Figure 10 (M5). In some cases, the center of the spans coincides with the center of an alveolus. In these cases, in addition to the beam with the stiffener in the center, another beam with stiffeners in the web posts adjacent to the center of the span was analyzed, as shown in Figure 11 (M1).

The beams subjected to a concentrated load (M7 a M14), with stiffeners placed only in the point of load application and in the supports, suffered web post buckling by shearing, in the web posts without stiffeners closer to the supports. Versions of these beams with stiffeners in the web posts close to the supports were analyzed as well, as shown in Figure 12.

Tabla 1. Characteristics of the analyzed beams

beams	pattern	h_p (mm)	number of openings	load type
M1	AS	-	7	uniformly distributed load along the entire span
M2	AS	100	7	
M3	PN	-	5	
M4	PN	100	5	
M5	LT	-	4	
M6	LT	100	4	
M7	AS	-	7	concentrated load at the midspan
M8	AS	100	7	
M9	PN	-	5	
M10	PN	100	5	two concentrated loads in the middle thirds of the span
M11	AS	-	7	
M12	AS	100	7	
M13	PN	-	5	
M14	PN	100	5	

Notes: AS – Anglo-Saxon; LT – Litzka; PN - Peiner

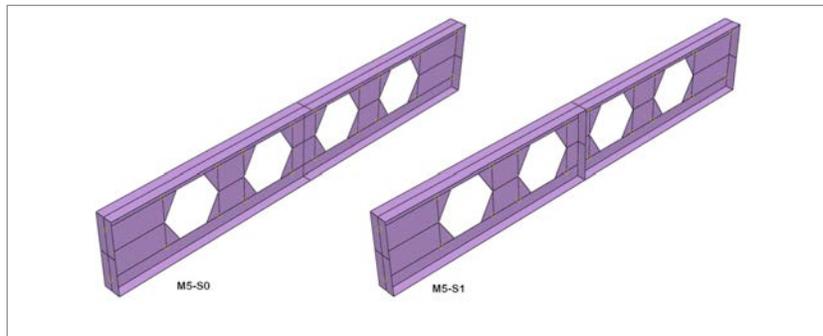


Figure 10. M5 beam, with and without additional stiffener

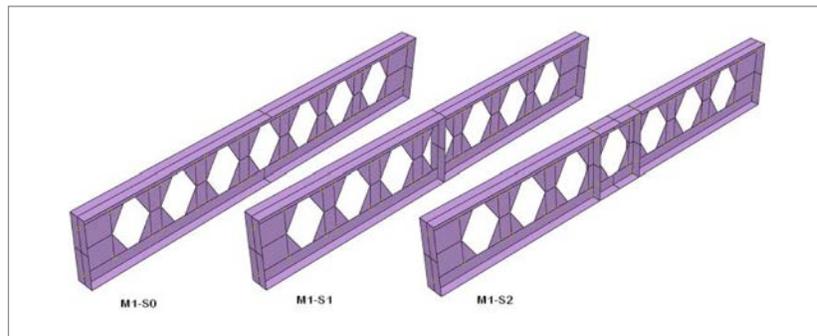


Figure 11. M1 beam, without and with (one and two) additional stiffeners

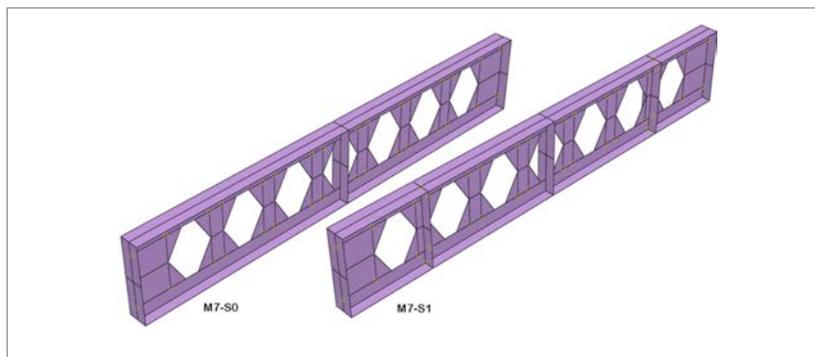


Figure 12. M7 beam, without additional stiffener and with two symmetrically placed stiffeners

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Table 2, Table 3 and Table show the results for the beams' ultimate capacity. The names of the beams are followed by the S0, S1 or S2 terminology, that corresponds

respectively to the beams without an additional stiffener, with one stiffener and with two stiffeners.

Table 2. Ultimate capacity and failure mode in the analyzed beams – distributed load

Beam	Failure mode	Ultimate load (kN/m)	Gain
M1-S0	WPB by compression	107,8	-
M1-S1	WPB	128,3	19%
M1-S2	WPB	129,3	20%
M2-S0	WPB by compression	70,7	-
M2-S1	WPB	99,1	40%
M2-S2	WPB	101,4	43%
M3-S0	WPB by compression	118,6	-
M3-S1	Vierendeel; WPB	132,1	11%
M3-S2	WPB; Vierendeel	135,6	14%
M4-S0	WPB by compression	84,3	-
M4-S1	WPB	113,5	35%
M4-S2	WPB	115,6	37%
M5-S0	WPB by compression	121,3	-
M5-S1	Vierendeel	141,7	17%
M6-S0	WPB por compressão	86,6	-
M6-S1	WPB	134,4	55%

Table 3. Ultimate capacity and failure mode in the analyzed beams – concentrated load

Beam	Failure mode	Ultimate load (kN/m)	Gain
M7-S0	WPB	266,7	-
M7-S1	WPB	281,1	5%
M8-S0	WPB	182,2	-
M8-S1	WPB	201,7	11%
M9-S0	WPB	269,1	-
M9-S1	Vierendeel	280,8	4%
M10-S0	WPB	222,0	-
M10-S1	WPB	257,5	16%

Table 4. Ultimate capacity and failure mode in the analyzed beams – two concentrated loads

Beam	Failure mode	Ultimate load (kN/m)	Gain
M11-S0	WPB	144,9	-
M11-S1	Vierendeel	189,3	31%
M12-S0	WPB	120,0	-
M12-S1	FMA; Vierendeel	165,8	38%
M13-S0	WPB	144,0	-
M13-S1	Vierendeel	149,6	4%
M14-S0	WPB	124,8	-
M14-S1	Vierendeel	149,4	20%



6. Discussion of the results

The following figures present the load-displacement curves of each group of beams, with the purpose of showing the increase in resistance capacity. It is possible to notice that the stiffness in the elastic range does not change, since the stiffener works only as lateral restraint for the web posts subjected to any instability.

With the addition of web post stiffeners in the M1 beams (Figure 13), it is possible to observe an increasing of bearing capacity in 20% if compared with the one without stiffeners. Web post buckling occurs in the M1-S0 beam, with

change of the vertical displacement direction in the monitored point, due to the strains occurred (Figure 8). In beams M1-S1 e M1-S2, there is an occurrence of web post buckling by shearing, with maximum load values to close. That indicates that the presence of the stiffener in the M1-S1 beam, positioned in the alveolus center, is enough to prevent lateral displacements in the central part of the beam and, consequently, to avoid the occurrence of WPB by compression. In the M2 beams (Figure 14), the presence of stiffeners caused an increase of about 40% of the bearing capacity, showing a similar behavior in relation to the M1 beams.

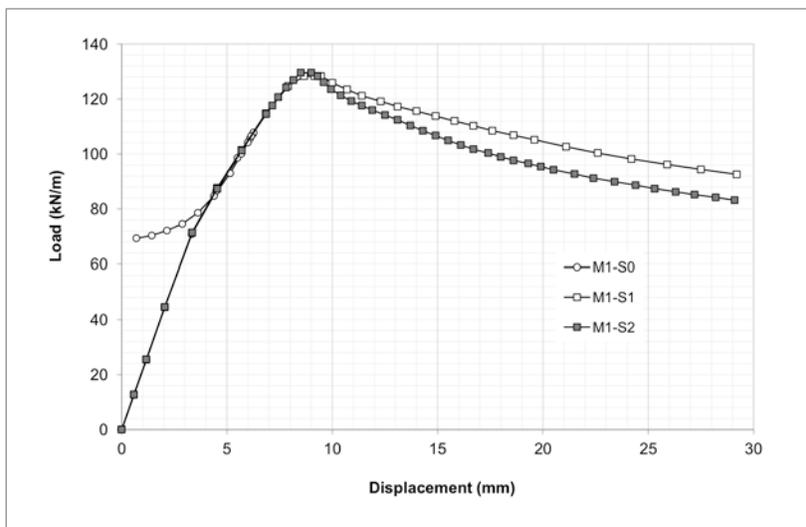


Figure 13. Load-displacement curves for the influence of web stiffeners in M1 beams

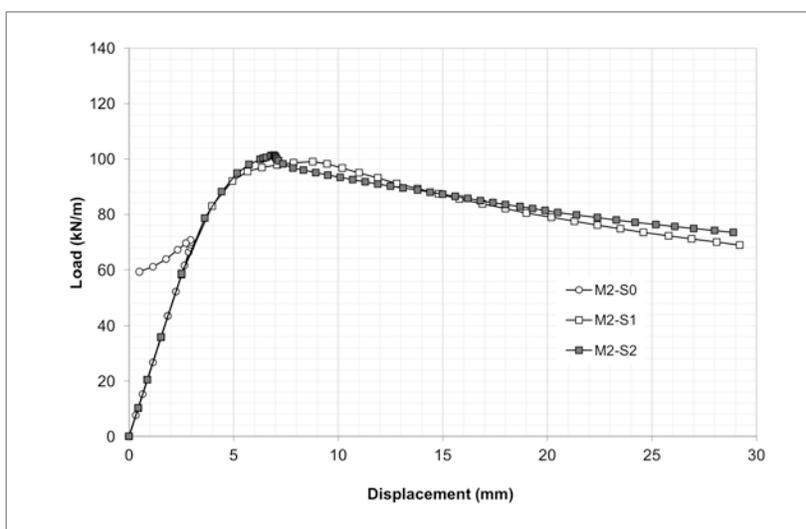


Figure 14. Load-displacement curves for the influence of web stiffeners in M2 beams

In beams M3 to M6 (Figura 15 to Figura 18), it is possible to observe a behavior that is similar to the previously analyzed beams, with significant increase in the ultimate capacity when using web stiffeners that prevent the occurrence of WPB by compression. Similarly, the beams whose stiffener in the center of the span coincided with the

center of an alveolus obtained a bearing capacity that is similar to the ones with two stiffeners placed in the center of the central web posts. It is possible to observe that the increase of resistance is more significant in beams where the web is more slender (beams with expansion plates).

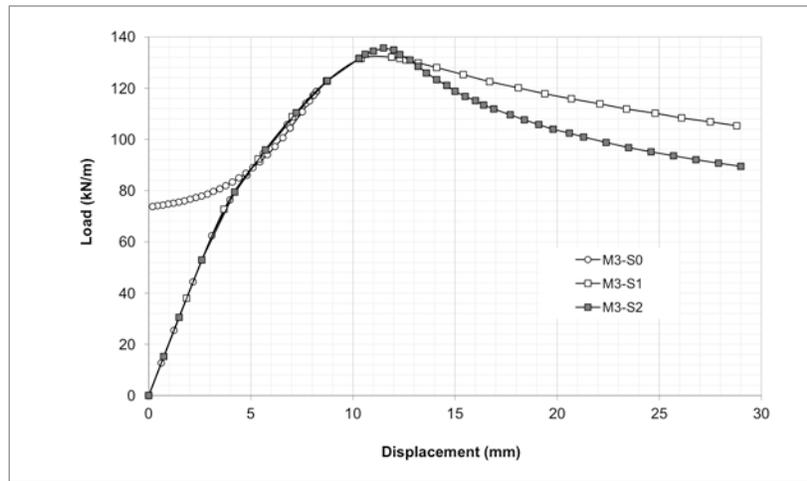


Figure 15. Load-displacement curves for the influence of web stiffeners in M3 beams

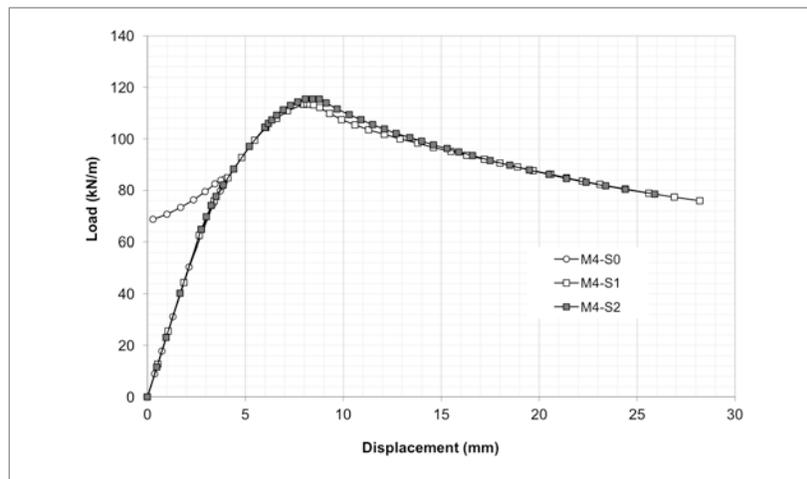


Figure 16. Load-displacement curves for the influence of web stiffeners in M4 beams

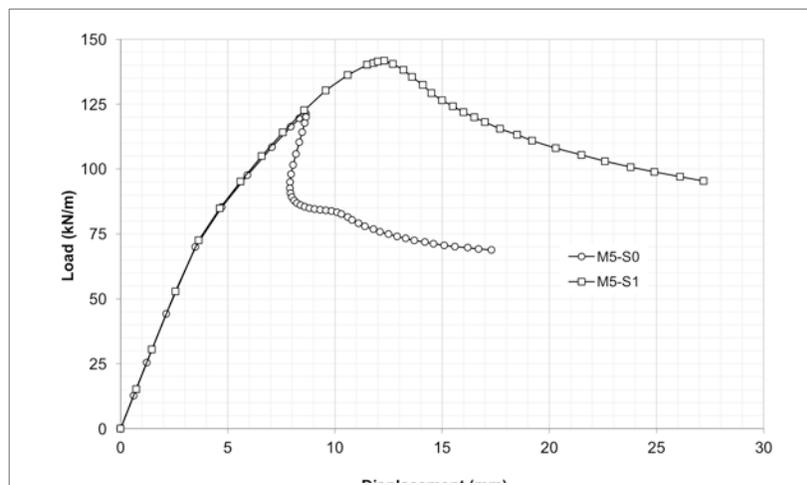


Figure 17. Load-displacement curves of the influence of web stiffeners in M5 beams



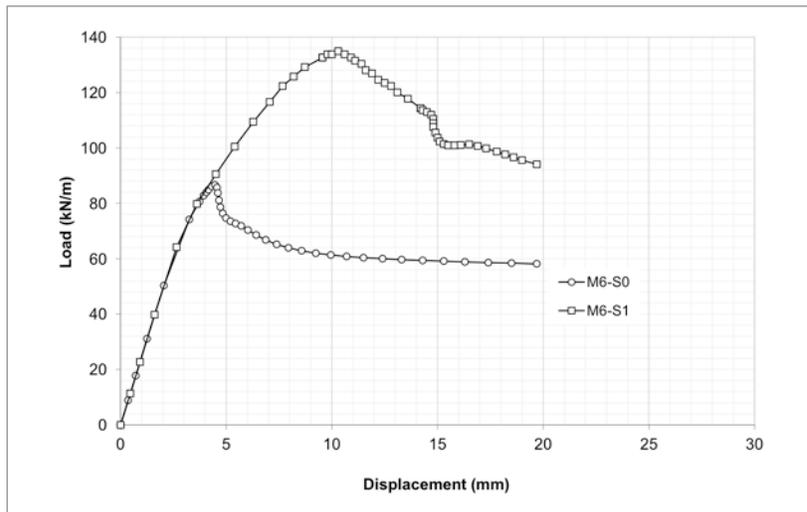


Figure 18. Load-displacement curves for the influence of web stiffeners in M6 beams

In beams M7 to M10 – subjected to a concentrated load in the center of the span – and in beams M11 to M14 – subjected to two concentrated loads in medium thirds of the span – the prevailing failure mode in the beams without

additional stiffeners was web post buckling by shearing. With the addition of stiffeners in the critical web posts, it was possible to observe an increase in the bearing capacity of the beams (Figure 19 to Figure 26).

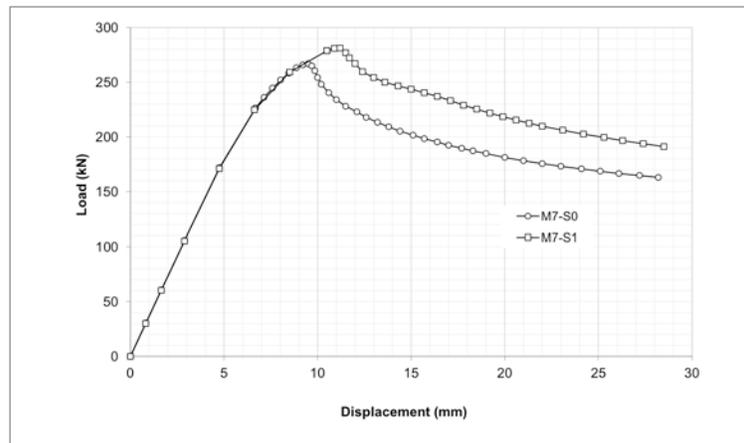


Figure 19. Load-displacement curves for the influence of web stiffeners in M7 beams

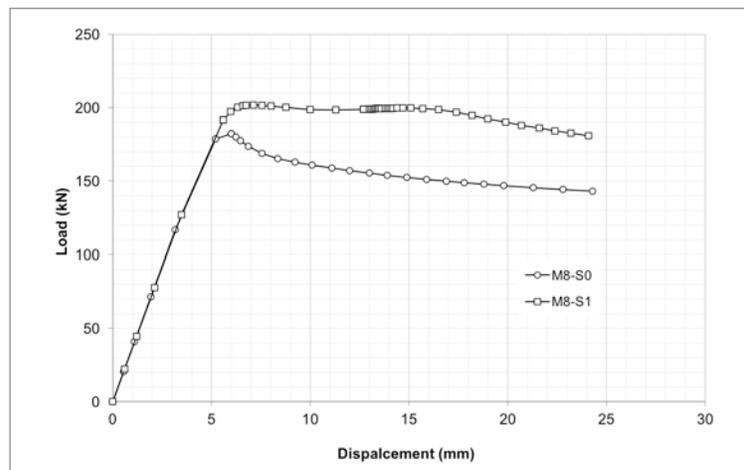


Figure 20. Load-displacement curves for the influence of web stiffeners in M8 beams

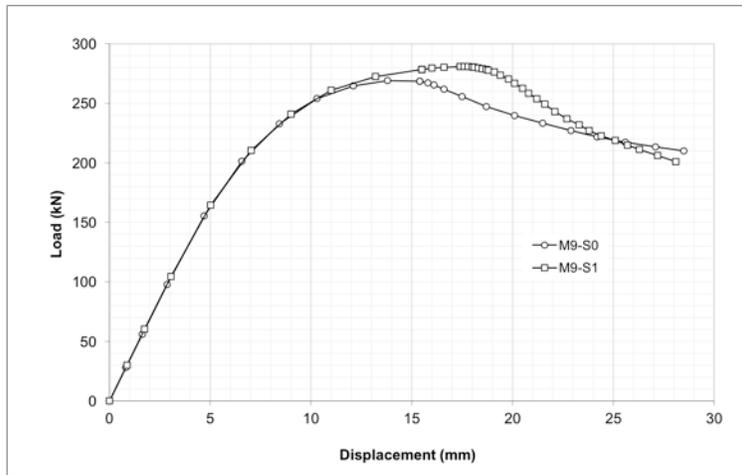


Figure 21. Load-displacement curves for the influence of web stiffeners in M9 beams

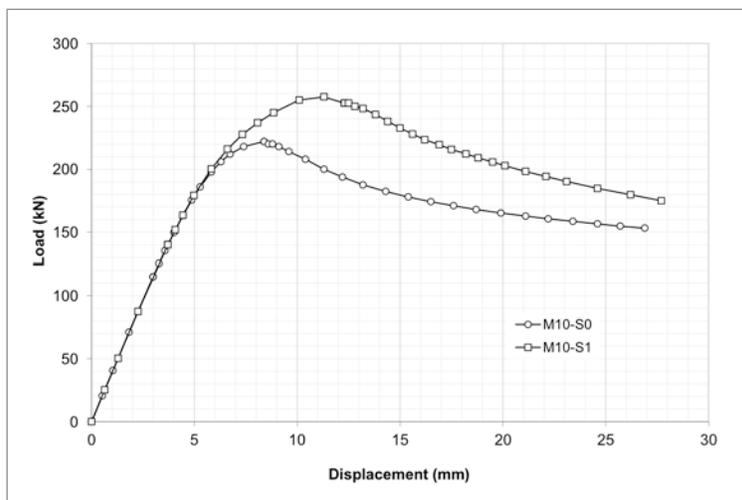


Figure 22. Load-displacement curves for the influence of web stiffeners in M10 beams

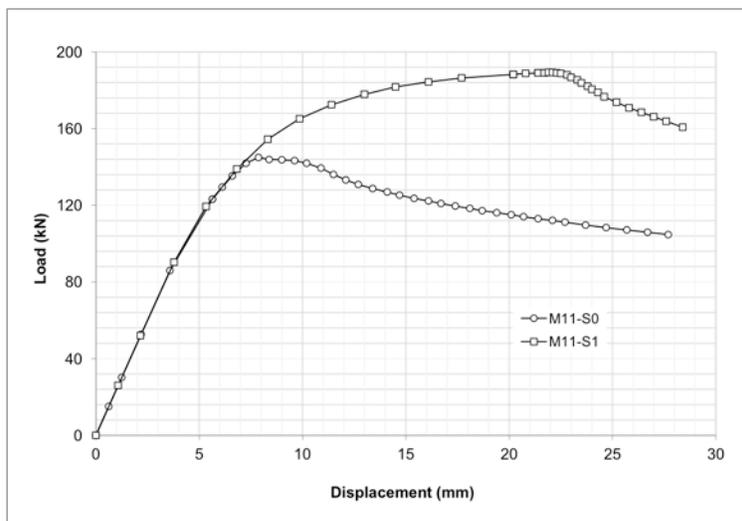


Figure 23. Load-displacement curves for the influence of web stiffeners in M11 beams

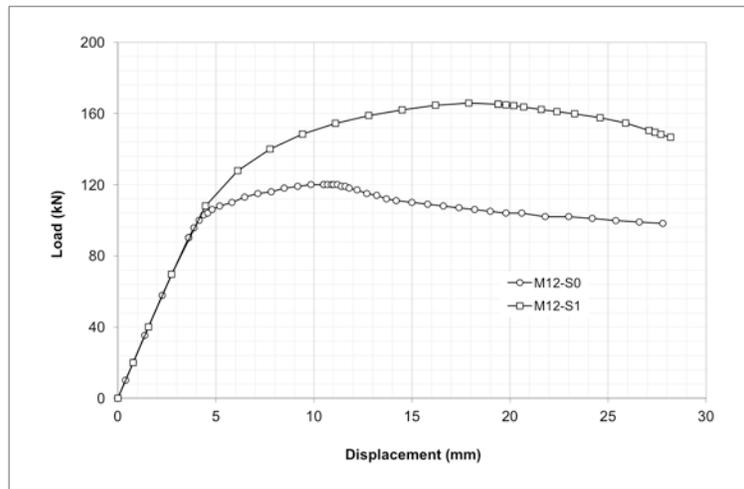


Figure 24. Load-displacement curves for the influence of web stiffeners in M12 beams

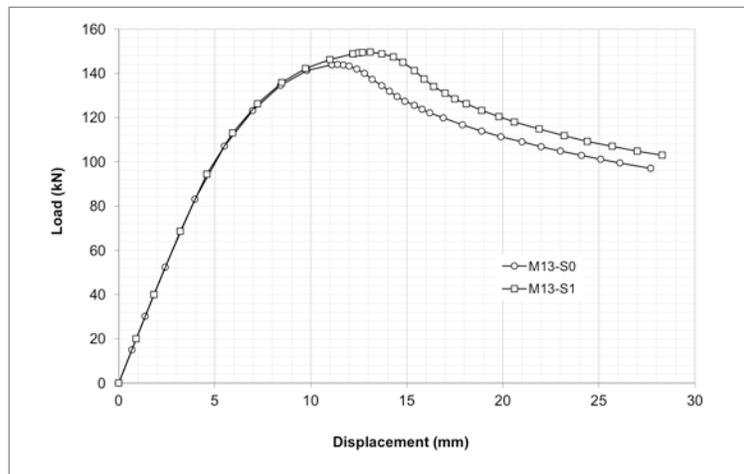


Figure 25. Load-displacement curves for the influence of web stiffeners in M13 beams

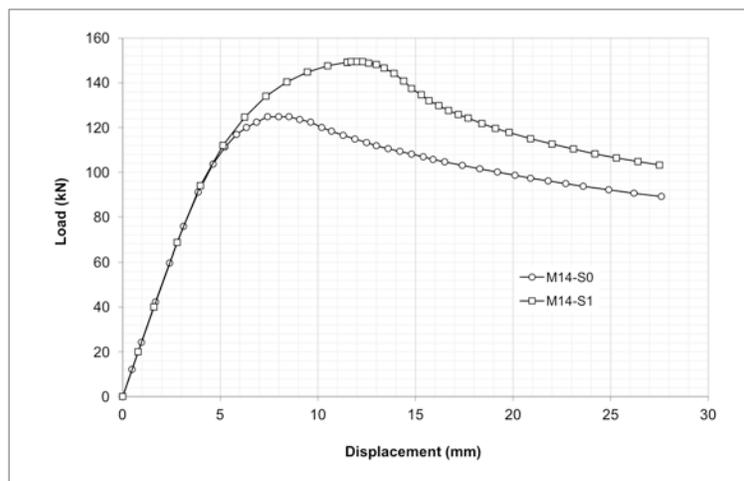


Figure 26. Load-displacement curves for the influence of web stiffeners in M14 beams

The average increase of bearing capacity in the beams subjected to Concentrated load was inferior to the ones with distributed load. This happens because, in the beams with distributed load, the web post buckling occurs by compression and, when applied, the stiffener works under axial compression, creating a cruciform section with the beam web. In the case of concentrated load, the force that causes web post buckling is in the horizontal direction, making the stiffener work under bending.

7. Conclusions

In beams with webs of small thickness, subjected to uniformly distributed load, the tendency is the occurrence of web post buckling close to the support region, caused by the interaction of normal and shear stresses in this region. If the beam is too short, and the applied loading gets close to its ultimate capacity, there is the possibility of buckling by compression of the web post close to the center of the span. In these cases, the presence of a web stiffener in the support region produces a favorable effect, helping to avoid the web post buckling close to the supports. In the simulations that were carried out, the addition of a stiffener with these characteristics in the beam's free span showed the beneficial effect of this stiffener, increasing the bearing capacity of the beam.

It is possible to observe that the web post buckling by compression was avoided with the placement of the stiffener in all beams, and that the failure by web post buckling by shearing occurs in the region of maximum shear. In the beams

where web post buckling by shearing occurs, the addition of stiffeners also increases the ultimate load and, in most cases, avoids the occurrence of the web post buckling, with the failure happening by the Vierendeel mechanism.

It is possible to observe in the load-displacement curves presented in this work that the beam's behavior in the elastic range is identical for the beams with or without additional stiffeners, since these elements only work as lateral restraint for the web posts subjected to instability.

Due to the way that the stiffeners behave in the beams subjected to distributed load and with web post buckling by compression, a higher effectiveness of the stiffeners was observed, increasing the bearing capacity up to 50% in relation to the beams without additional stiffeners.

In the beams with more slender web posts (with expansion plates) there is a superior increase of the bearing capacity, since the stiffeners stiffen the web post, moving away the possibility of elastic buckling.

It was also possible to observe, in the conducted analysis, that the presence of a stiffener, even if placed in the center of an alveolus, is enough to avoid the lateral displacement of the web posts in that region.

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9. References

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