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Diversity of Mathematical Learning Opportunities Within Chilean Kinder Classrooms of Different Socioeconomic Status

Diversidad de oportunidades de aprendizaje matemático en aulas chilenas de kínder de distinto nivel socioeconómico

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Abstract

In the last decade, scientific literature on early learning has consistently considered mathematics as a fundamental area of development in the early years. Nevertheless, we know very little about the characteristics of mathematics teaching in early childhood education in Chile. The goal of this study was to describe the teaching of mathematics in 14 preschool classrooms of different socioeconomic status with regards to the time dedicated to mathematics and the specific contents covered. Our results show that approximately 10% of the daily routine in preschool is used for mathematics, and that the topics covered are mostly counting, cardinality, and basic operations. Topics related to geometry, problem-solving, and mathematical language are scarce and more frequent in classrooms of a high-socioeconomic status. These results are analyzed in light of recent knowledge about mathematics education in early childhood.

Keywords: early childhood, mathematics, socioeconomic, status teaching.

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Resumen

In the last decade, scientific literature on early learning has consistently placed mathematics as a fundamental area of development in the early years. Nevertheless, we know very little about the characteristics of mathematics teaching in early childhood education in Chile. The goal of this study was to describe the teaching of mathematics in 14 kindergarten classrooms of different socioeconomic status with regards to the time dedicated to mathematics and the specific contents covered. Our results show that approximately 10% of the daily routine in kindergarten is used for mathematics, and that the topics covered are mostly counting, cardinality and basic operations. Topics related to geometry, problem-solving and mathematical language re scarce and more frequent in high-socioeconomic status classrooms. These results are analyzed in light of recent knowledge about mathematics education in early childhood.

Palabras clave: educación parvularia, enseñanza, matemáticas, nivel socioeconómico.

Introduction

Over recent decades there has been substantial progress in research on the development of mathematical skills in early childhood (Baroody, Lai, & Mix, 2006; Charlesworth, 2015; Cross, Woods, & Schweingruber, 2009; Sarama & Clements, 2009). The accelerated emphasis on this field of knowledge is largely due to consistent evidence that mathematical skills develop from the earliest years and that, if supported appropriately, they have the power to influence future school performance positively (Clements & Sarama, 2011; 2013; Clements, Sarama, Wolfe, & Spitler, 2013; Hachey, 2013; Sarama & Clements, 2009). Moreover, research shows that high-quality teaching practices in mathematics during preschool education or early childhood education not only increase children's mathematical skills, but can even be positively transferred to other areas such as language, self-regulation, or executive functions (Clements, Fuson, & Sarama, 2017; Clements et al., 2013; Farran, Lipsey, & Wilson, 2011; Sarama, Clements, Wolfe, & Spitler, 2012; Sarama, Lange, Clements, & Wolfe, 2012).

However, despite the importance of early development of mathematical skills, the literature reports poor quality teaching of this subject during the early years. In fact, the international evidence is revealing, as it shows that early education teachers have generally not been prepared with the appropriate level of knowledge to support the mathematical development of the children in their classrooms (Copley & Padron, 1998). Although most of the children build informal mathematical ideas early on (Baroody, 2009), it is particularly important for children of low socioeconomic status with fewer opportunities for mathematical learning to have access to high-quality preschool education, which provides them with the informal and formal mathematical experiences that they do not always receive at home (Clements & Sarama, 2011; Jordan, Huttenlocher, & Levine, 1992).

In this context, the question about the mathematical learning opportunities to which children of different socioeconomic status have access during preschool education is essential if we are interested in improving mathematics skills at all educational levels in the country.

Theoretical background

Theories about development of mathematical skills in the initial stage of education have moved on from an approach where children had nothing to learn, to theories that have positioned mathematics as one of the priority areas of cognitive development in the early years (Baroody et al., 2006; Cross et al., 2009; Geary, 2006; Sarama & Clements, 2009; Saxe, Guberman, & Gearhart, 1987).

Starting with the seminal work of Jean Piaget on learning the concept of numbers, current theories have consistently demonstrated that mathematical skills develop from birth, through a complex interaction between innate abilities, experience, and language (Baroody et al., 2006; Geary, 2006; Sarama & Clements, 2009).

New theories on learning mathematics in early childhood have had a marked impact on what a preschool teacher should teach and how they should lead this process. In fact, the National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC), both based in the United States, have issued statements about the importance of including mathematics in early education and the need to include resources to strengthen these skills in the early childhood classroom (Clements, Copple, & Hyson, 2002).

Quality mathematics education in preschool education

A recent review by Clements et al. (2017) identified the most effective practices for teaching mathematics in early childhood settings. These good practices have been recommended in various joint publications by experts in mathematics and early childhood (NCTM and NAEYC) and can be summarized in four main elements. Educators who teach high quality mathematics:

a. Trust in the children's abilities and support them to make sense and “mathematize” the real world. For example, they provide scenarios that connect specific mathematical language and symbols with quantities and actions in the real world; they direct children's attention towards the crucial aspects of the mathematical ideas on which they are working, to help them make connections with other mathematical ideas; and they design multiple experiences that give children the time and opportunity to develop their ideas, deepen their understanding, and improve their mathematical fluency.

b. Create an environment where one talks and thinks about mathematics, an environment that reinforces, nurtures, and pays attention to children's mathematical thinking to help them explain themselves and to help each other explain and resolve mathematical problems.

c. Plan their mathematical teaching considering the development trajectories of children's mathematical thinking.

d. Include real three-dimensional objects, drawings, and other two-dimensional representations in their teaching to help children make sense of mathematical structures and use the drawings as representations of their mathematical thinking.

In this vein, intervention plans have been developed that have been intended to improve the practices developed by early childhood teachers in their curricular plan (Clements & Sarama, 2007; Greenes, Ginsburg, & Balfanz, 2004; Griffin, Case, & Siegler, 1994). These interventions have shown that the effect of providing enriched environments in mathematics during preschool education has an impact on improving the mathematical skills of the participating children (Clements & Sarama, 2011; 2013; Clements et al., 2013; Sarama & Clements, 2009).

However, the evidence suggests that early childhood teachers generally demonstrate low knowledge of what the most effective pedagogical practices are for the development of mathematical skills (Carpenter, Fennema, Peterson, & Carey, 1988; Sarama & Clements, 2009) and base their teaching on activities that only tangentially address mathematical content. For example, they provide "mathematical" materials such as Lego, beads, or blocks during times of transition, but without a clear pedagogical intent; they repeatedly copy numbers without a context of what they mean; they have children draw or "fill in" numbers or geometric shapes solely as motor activity; or they memorize the names of certain geometric figures without examining their attributes, among other actions. Although these practices are widespread in early childhood classrooms (Sarama & Clements, 2009), it has been shown that they are mostly ineffective in teaching mathematics, due to their lack of explicit connection between mathematical concepts and procedures, in addition to a lack of intentionality to mediate between "doing" and children's mathematical knowledge (Cross et al., 2009).

At present, there is scant national evidence that allows us to find out what happens in the preschool classroom during teaching of mathematics. In this respect, the study by Strasser, Lissi and Silva (2009)—which looked at the time allocated in 12 classrooms of different socio-economic status (SES)—showed that mathematics activities take up 9% of the teaching time, that is, about three minutes of a full day. In addition, these findings are independent of the socioeconomic characteristics of the institution.

On the other hand, in a study of classroom observation and discourse analysis in focus groups, Ormeño, Rodríguez, and Bustos (2013) found that, although preschool educators recognized the importance of mathematical education in the early years and had great appreciation of the discipline, they perceived themselves as having weaknesses and being ignorant about the teaching strategies they should use, how to organize the space, and what children should know at this level.

A study by Ponce, Reyes, and Lamig (2017), which investigated the beliefs of 12 students in the final year of a Preschool Education degree at four universities in Santiago, Chile, revealed that students attach high importance to mathematics teaching, but they project their own future pedagogical practices as "spontaneous" (unplanned) and "integrated" with other curricular areas (not explicit), arguing that children essentially learn mathematics by participating in the context of the classroom and without there necessarily being explicit teaching of these skills.

The aim of this study is to describe the characteristics of mathematics education used by a group of preschool educators belonging to 14 preschool classrooms of different SES in the capital of Chile and to reflect on their suitability based on the current recommendations for teaching in early childhood reviewed in specialized literature. The characteristics of mathematical teaching are described based on the variables: time dedicated to the teaching mathematics in daily work; actions and content addressed in the segments of the day during which work is done on this topic; and the SES of the educational center.

Methodology

Participants

A total of 27 videos were reviewed from 14 classrooms of the highest transition level of preschool at five educational centers in Santiago with different SES. The preschool courses in the country serve children between the ages of five and six and this is the last level of preschool education before they enter elementary education.

Of the 14 classrooms that participated, 13 were filmed twice during the second half of 2014 and one was recorded only once during the same period of time. The characteristics of the classrooms included in the study are shown in Table 1.

Table 1. SES, number of classrooms and total number of children by education center

Centro educativo	NSE del centro educativo (según datos Simce[1])	Número de aulas	Total niños
Center_1	Low	6	109 (35.5%)
Center_2	Low	2	57 (18.57%)
Center_3	Medium-high	2	45 (14.66%)
Center_4	Medium-high	2	37 (12.05%)
Center_5	High	2	59 (19.22%)
Total		14	307 (100%)

Source: Prepared by the authors.

Recording procedure

During the first semester of 2014, the management of various institutions in the city of Santiago that provided preschool education were contacted by email. Of the 40 institutions that expressed an interest in taking part in the study, five education centers were selected based on their socioeconomic characteristics and the feasibility of data collection (for example, the availability of dates to visit the institution, the number of children per level, availability for video recordings, and willingness of families to participate in the study, among other criteria). All of those responsible for the educational institutions in this study gave written consent for their institution to participate, and both the preschool educator responsible for the level and the families of the children in the classroom were informed and also gave written consent for either their participation or that of their children.

The video recordings made correspond to a full work day (one full morning). The arrival time at the establishment coincided with the entry time of the children, which ranged between 7.50 a.m. and 8.30 a.m. and lasted until noon, in an interval that included between 12.30 p.m. and 13.00 p.m. approximately.

The video recording time was focused on the preschool educator, which implies that the recordings did not include the times in the day when the children were under the charge of another adult (for example, technical personnel in preschool education, or teachers in areas such as physical or religious education, among others), but they did include all the times when the preschool educator was in charge of the group, including times in which educational work was done and transition times, such as periods of recreational time, bathroom breaks, or for eating.

All of the recordings were done by preschool education students in the third year of their degree course.

Coding system

The procedure to create the coding guidelines used in this study involved various steps. First, we revised the frameworks of the national (Ministerio de Educación de Chile, Mineduc, 2001) and international curricula (Sarama & Clements, 2009), in which we identified the main content addressed at the level and, using this information, we created the first set of codes. Subsequently, this code set was compared with the review of three videos of full days in preschool classrooms taking part in another study. During this phase the codes were reviewed and adjusted and examples and specific descriptions of each code were included, along with codes for

1. National system of assessment of learning outcomes.

actions not initially considered. Finally, the complete coding guideline was reviewed by experts in mathematics and early childhood education, who made suggestions and specifications that were used to configure the final version of the instrument for this study.

The system used included 16 codes that account for the actions and content that preschool educators use most frequently in teaching mathematics in preschool education. These codes were grouped into eight dimensions of mathematical work: counting and cardinality, basic operations, numerical decomposition, measurement and data, geometry, problem solving, definition of mathematical concepts, and “pre-numerical” activities. The total time of mathematical work done in each classroom and the duration of each work segment were also quantified.

Coding

Two trained coders—a Master's degree student in Educational Psychology and the lead investigator—participated in the process of coding the video recordings. In this process each coder reviewed the videos independently, identifying the segments of the day in which work was done with mathematics. When they identified a segment that lasted more than 30 seconds, the coder marked the start and end time. Subsequently, they described the presence or absence of each of the codes in the coding scheme in that workspace. Each workspace could include more than one code in the proposed system; for example, if an educator posed a mathematical problem that involved counting and writing numbers in a work segment, this was coded at the start and end time and using the codes indicated: problem solving, counting, and writing numbers.

In a subsequent stage, both coders met and compared their results. Coincidence between them on the final decision of each coder was considered as an agreement, that is, both indicated whether or not the code was present in the segment. To calculate the agreement in the duration of each workspace, an error of up to five seconds was considered, both at the beginning and the end of the segment.

Of the 27 days recoded, 14 were double-coded, which included 38 of the 49 mathematical work segments identified in this study, achieving final agreement on 80% of all codes evaluated, with a range of 81.25% to 100% agreement on each code, and average general agreement of 97.14% on all codes. The agreement between coders for the identification of the start and end of the work segments was 81.58%. For each double-coded video recording, the coders together established a final version of the coding. Subsequently, each coder respectively analyzed seven and six videos individually, obtaining the final codes used in this study.

Table 2 summarizes the observed dimensions, the codes used, and examples of these in the video analysis guideline.

Table 2. Dimensions observed, their definition, codes used, and examples of activities in each dimension.

Dimension	Definition	Codes	Examples of activities
Counting and cardinality.	Verbally counting a set of objects, representing the quantity of a group of objects with a written number, and/or comparing quantities and numbers.	1. Counting. 2. Representing a series of objects with a written number. 3. Comparing quantities.	• Counting a set of objects shown in order and disorder. • Counting objects and writing the total number. • Counting and comparing quantities in a group of objects in relation to another group of objects. • Identifying the larger, smaller, or equal number, when comparing two numbers.
Basic operations.	Representing and/or solving addition or subtraction with objects, algorithms, or with both.	4. Representing and/or solving an addition or subtraction with elements. 5. Adding and subtracting using algorithms.	• Using specific material to add or subtract. • Writing an addition or subtraction with the algorithm that represents it (e.g.: 8 + 4 = 12).
Numerical decomposition.	Composing and/or decomposing numbers in their decimal composition (unit, tens, hundreds), using objects, drawings, or algorithms.	6. Composing and/or decomposing numbers. 7. Registering a composition or decomposition using an algorithm..	• Joining 10 objects to show a 10 and using unique objects as units. • Registering a composition or decomposition through an algorithm (e.g., 18 = 1 dozen and 8 units).
Measuring data.	Describing or comparing objects based on their characteristics, such as, for example, length or weight. Also includes activities that that imply ordering some kind of information in graphic representations.	8. Describing and comparing by characteristics. 9. Graphic representations.	• Describing objects based on their characteristics: "This pen is long", "the table is very heavy", etc. • Comparing objects with a common measurable attribute ("more than", "less than"). • The number of children who have pets and of what kind is counted, and then a bar graph is created with that information.

Source: Prepared by the authors.

Table 2. Dimensions observed, their definition, codes used, and examples of activities in each dimension.

Dimension	Definition	Codes	Examples of activities
Geometry.	Identifying, describing, and comparing two- and three-dimensional geometric shapes based on relevant attributes such as number of sides, vertices, or faces. Describing the relative position of an object using terms such as: up, down, beside, in front, behind, to the left, to the right.	10. Naming shapes.. 11. Describing shapes. 12. Comparing shapes. 13. Spatial position.	• Naming and constructing geometric shapes using various materials (e.g. sticks and balls of clay). • Describing and comparing objects in the surroundings using geometric shapes (e.g. the clock is a circle, the blackboard is a rectangle, etc.). • Identifying an object based on its position in space. For example, "the pens are beside the yellow shelf and behind the table".
Problem solving.	Solve an unknown presented through a mathematical problem that has a statement that places the information in a real-life context.	14. Problems.	• Andrea, Juan, and Felipe live in a building with 10 floors. Andrea lives on floor 5, Juan on floor 8, and Felipe on floor 2. How many floors is Andrea from Juan and from Felipe? • The educator wants to know how many children attended today to request breakfast for everyone. The preschool educator asks the children to develop strategies to find the solution.
Definition of mathematical concepts.	The educator defines, explains, or exemplifies mathematical concepts, which may or may not be used to solve a problem, incorporated into the context of a mathematical task, or just used to expand mathematical language.	15. Definition.	• "Children, look at what Fernanda has constructed! A pattern! It is a pattern because there is one red and one blue, one red and one blue …" • "What can we do in mathematics corner? We can count the cubes to make a house… We can add… adding is when we put objects together". • Daniel has 10 pens! What is that called? A ten, because there are 10 objects.

Source: Prepared by the authors.

Table 2. Dimensions observed, their definition, codes used, and examples of activities in each dimension.

Dimension	Definition	Codes	Examples of activities
"Pre-numerical" activities	Activities that imply carrying out procedures.	16. Pre-numerical activities.	• Copying a number several times following a model of points. • Singing songs. • Perception activities (e.g. figure and background).

Source: Prepared by the authors.

Results

Duration of the day and time dedicated to mathematical work

In this study, the 27 days observed had an average duration of 147.39 minutes (SD = 30.61). The shortest day lasted 93.22 minutes and the longest 187.55 minutes.

In the days observed, an average of 14.98 minutes were devoted to mathematical work (SD = 11.64), in a period ranging from 2.17 minutes to 51.55 minutes, which represents an average of 9.7% of the total duration of the days.

In the video recordings reviewed, 49 segments with mathematical work were identified and counted with an average of 3.5 (SD = 1.29) work segments each day, ranging between 1 and 6 segments per day. The average duration of these segments was 14.29 minutes (SD = 15.77).

Contents identified in the mathematical work segments

Each of the segments with observed mathematical work was assessed using the set of codes explained previously. Table 3 summarizes the average and standard deviation of the relative frequency of each of the codes in the dimensions assessed in all classrooms included in the sample.

Table 3. Average and standard deviation of the relative frequency of each code in the mathematical work segments assessed (codes are not mutually exclusive)

Codes and dimensions	Average	Standard deviation
1. Counting	0.74	0.28
2. Representing a series of objects with a written number	0.34	0.32
3. Comparing quantities	0.28	0.32
Average frequency of counting and cardinality	0.45	0.17
4. Representing and/or resolving addition or subtraction with elements	0.52	0.28
5. Adding and subtracting using algorithms	0.37	0.28
Average frequency of basic operations	0.44	0.23
6. Composing and decomposing numbers	0.12	0.17
7. Registering a composition or decomposition using an algorithm	0.04	0.09
Average frequency of numerical decomposition	0.08	0.11
8. Describing and comparing by attributes	0.05	0.10
9. Graphic representations.	0.05	0.14
Average frequency of measuring and data	0.05	0.08
10. Naming shapes	0.00	0.00
11. Describing shapes	0.02	0.06
12. Comparing shapes	0.00	0.00
13. Spatial position	0.00	0.00
Average frequency of geometry	0.00	0.01
14. Mathematical problems	0.20	0.24
Average frequency of problem solving	0.20	0.24
15. Definition	0.13	0.24
Average frequency of definition of mathematical concepts	0.13	0.24
16. Pre-numerical activities.	0.07	0.13
Average frequency of pre-numerical activities	0.07	0.13

Note: The relative frequency is equal to the number of work segments in the day divided by the frequency of occurrence of each code in each segment (n = 49).

Source: Prepared by the authors.

The main results show that counting and cardinality are included in the contents that are most frequently used in the classroom, present in more than 70% of the work segments observed. The codes associated with cardinality are seen in smaller quantities, but they are still the codes with the greatest presence in the reviewed work segments.

Similarly, the content related to basic operations is ranked second in frequency in the classrooms evaluated. Some 52% of the segments are devoted to working on operations such as addition and subtraction with specific material, and about one third of the segments concentrate on working on algorithms that describe addition and subtraction.

On the other hand, activities regarding the knowledge of numbers and their composition and decomposition, as well as activities where mathematical concepts are defined are conducted in about 10% of the total mathematical segments assessed.

Comparisons by SES

For this comparison, the SES of the education centers was constructed considering the categorization provided by Simce for each education center and grouping the classrooms of medium-high and high SES, producing two groups, one consisting of eight classrooms (166 children) for low SES and six classrooms (141 children) for medium-high and high SES. The results of these comparisons are shown in Table 4.

Table 4. Student's t test for independent samples in each of the variables under study according to the SES of the education center

Variable	Low SES		High SES		Student's t
	Average	Standard deviation	Average	Standard deviation	
Total duration of the day (min.)	146.90	29.07	147.96	32.42	-0.315
Duration of mathematics work (min.)	10.31	4.10	**20.51**	14.83	**-8.271****
Percentage of mathematics work	6.86	2.35	**13.06**	9.67	**-7.768****
Counting and cardinality	**0.47**	0.17	0.43	0.17	**2.048***
Basic operations	0.40	0.17	**0.49**	0.28	**-3.348****
Numerical decomposition	**0.09**	0.12	0.07	0.09	**2.105***
Measuring and data	**0.06**	0.10	0.03	0.05	**3.69****
Geometry	0.00	0.00	**0.01**	0.02	**-6.46****
Problem solving	0.08	0.11	**0.35**	0.27	**-11.158****
Definition of mathematical concepts	0.06	0.12	**0.21**	0.32	**-5.613****
Pre-numerical activities	**0.09**	0.15	0.04	0.08	**3.982****

Note: *$p < 0.05$; **$p < 0.001$; $n = 49$.

Source: Prepared by the authors.

As Table 4 shows, we can observe significant differences according to SES in all the measured variables, with the exception of that measuring the total duration of the day, which is statistically the same in all the observed classrooms.

In terms of the time devoted to mathematical work and the percentage of the day that this represents, we observe that classrooms with high SES dedicate about 6% more time to mathematical work than the classrooms with low SES, which means that the children in these classrooms have about 10 minutes per day more mathematical work than children of low SES.

As regards the content on which work is done, classrooms that mostly serve children of low SES surpass classrooms with children of high SES in the amount of content related to counting and cardinality (counting, representing a quantity with a written number, comparing quantities), in activities that include content on numerical decomposition (composing and decomposing numbers with specific material or algorithms), in measurement and data activities (measuring attributes and graphic representations), and pre-numerical activities (classification, seriation).

On the other hand, classrooms that serve the population of high or medium-high SES have a higher frequency of activities on content associated with basic operations (adding or subtracting with elements and/or resolving addition or subtraction with algorithms), problem solving, and the definition of mathematical concepts than classrooms with children of low SES.

Analysis of observed mathematics teaching in classrooms during this study

A second aim of this study was to consider the suitability of mathematical teaching practices observed based on the current recommendations for early childhood education reviewed in the specialized literature. Given the frequency of activities addressing content related to the knowledge of numbers and basic operations, in addition to the theoretical relevance of the time devoted to mathematical work, we consider these three foci to analyze mathematical teaching in the classrooms observed.

Time spent on mathematics work. Similar research has shown that the time devoted to teaching of mathematics has been understood as a measurement of the opportunity for children to connect with mathematical knowledge (Sonnenschein & Galindo, 2015). In this study we observed that the average time dedicated to teaching mathematics in all the classrooms observed is short (approximately 15 minutes) and, in addition, it is even less in classrooms that serve children with low SES (about 11 minutes) compared to classrooms with children of medium-high and high SES (approximately 21 minutes). International recommendations based on evidence are that preschool classrooms should devote approximately one hour a day to working on mathematical ideas (Clements et al., 2017). This time may include work in large groups in areas such as the attendance register, the calendar, or metric recording of the weather; small group work in individual or collective experiences, such as problem solving, and work with manipulating materials in learning areas. While the recommendation on the time devoted to mathematical work in preschool education is applicable to classrooms of all socioeconomic levels, it is especially relevant for the most disadvantaged children, who should be exposed to more enriched mathematical experiences, whether in terms of their overall experience or in the quality of the experiences they receive (Sonnenschein & Galindo, 2015), which our results show does not occur in the classrooms observed.

Counting and cardinality. The specialized literature establishes that learning to count is one of the most important activities in children's early years, given its influence on subsequent mathematical learning (Baroody et al., 2006; Clements & Sarama, 2007; Dewey, 1976; Jordan et al., 2013). In the classrooms observed, counting is used in more than 70% of the segments on mathematical work and is the most widely used content in the classrooms included in this study. However, the experience of "counting" is extremely rich in opportunities to reflect on numbers and quantities, especially at the preschool level, and this was not generally observed in the participating classrooms. Our results show that in the classrooms observed, "choir" counting was emphasized, where the educator leads the counting activity and the children merely "chant" the numbers and counting of groups of equal objects, the quantities of which never exceeded 20 elements.

The specialized literature recommends that during the final years of preschool education children should participate in experiences enriched with counting, which allow them to expand the development of this concept. *Counting Collection* is an example of an enriched activity, outlined by researchers Schwerdtfeger and Chan (2007). This proposal emphasizes the importance of counting diverse groups of objects, especially those in which the unit to be counted is not evident. In these activities the educators give the children different groups of objects and they are initially only asked to select a group, count it together with a partner, and record which group was counted and the total.

Subsequently, the experience is made more complex by asking the children to show the educator how they counted, which provides space for discussion about counting strategies, or changing the type of objects that should be counted, such as going from counting a collection of buttons to a collection of buttons with different shapes, with different amounts of holes, or of different colors and sizes, among other variations. This counting, in which the unit to be counted is not evident and where there is reflection on what needs to be counted, is the counting that helps children classify, think flexibly, and make sense of the task of counting. In addition, this type of counting can give rise to thinking about other mathematical ideas such as patterns, mathematical operations, or measurement (Sarama & Clements, 2009).

Also, within the dimension of counting and cardinality there is extensive evidence, as has been summarized in studies such as those by Sarama and Clements (2009) and Fuson (1988), that during preschool children are prepared not only to count to 100 and establish the cardinality of a set of objects up to 30, but to also count backwards and forwards in small groups of objects (up to 10); in counting identify the pattern underlying the decimal number system (for example, identify the regularity of 11, 12, 13…; 21, 22, 23…; 31, 32, 33…, where the number that changes is the ten and the units are repeated in a stable manner); and identify the place value of the numbers, among others. This learning, typical of the dimension of counting and cardinality, was rarely observed in the video recordings reviewed and indicates a gap between what children are prepared to do and the opportunities they have to learn in the classroom.

Basic operations. During their initial years of education, children spend a great deal of time learning to operate with single-digit numbers. In this regard, Verschaffel, Greer, and De Corte (2007) point out that the discussion about what this knowledge means and, more importantly, how it is acquired, has undergone a major shift in perspective in recent decades. The authors argue out that for a long time the learning of simple addition and subtraction using a single digit was based on the repetition of these operations until the children had "memorized" them. However, the current approaches put greater emphasis on children's previous knowledge and the gradual development of these facts through strategies that are "informal" or invented by the children themselves (Baroody et al., 2006; Verschaffel et al., 2007). In the classrooms observed in this study, children frequently participate in segments of mathematical work where basic operations are included, either using manipulable materials or algorithms, but it was often observed that the teaching was focused on the repetition of decontextualized algorithms (for example, 5 + 7 = ?) or on finding the final result of an operation through a single strategy established as being valid (for example, reaching the total by counting all the elements in a sum). According to Verschaffel et al. (2007), learning the sum of single-digit numbers, for example, is developed through three overlapping phases. In Phase 1, children can solve small sums using simple counting strategies. For example, to solve the sum 2 + 3, children can count two fingers on one hand (one, two ...) and then count three fingers in the other hand (one, two, three), before finally counting all the fingers together (one, two, three, four, five) and thus concluding that the total is five. In Phase 2, children become more efficient in their counting strategies and begin to apply them to solve operations. For example, to solve 2 + 3 children can count from the last number (two ..., three, four, five), starting from the largest number (three ..., four, five) or "invent" combinations using facts that they already know, for example, 2 + 3 = [2 + 2 = 4] + 1 = 5. Finally, In Phase 3, children create "rules" that they can apply to any operation, such as knowing that, in a numerical sequence, the next number is always N plus one or that N + *10* = N*0*.

The trajectory of development of the operations indicated by the authors has a strong base in the experience of counting and the strategies that children use to make this counting more efficient (Phase 1), subsequently—and based on their previous experiences—the children will transition to more automated strategies that they themselves construct, such as those indicated in Phase 2 and 3. For children to acquire this type of knowledge they must have the opportunity to rehearse lots of counting strategies in different scenarios, to think about them and their effectiveness, and invent new ones, in addition to exercising these strategies for problems that make sense to them, which was rarely observed in the participating classrooms.

Discussion

The purpose of this study was to describe the characteristics of the mathematical teaching used by a group of preschool educators belonging to 14 preschool classrooms of different SES in the capital of Chile and to reflect on their suitability based on the current recommendations for early childhood education, reviewed in the specialized literature.

Firstly, this study showed that the time spent teaching mathematics in all participating classrooms is low. In fact, this research explicitly asked the educators of the participating preschools to conduct mathematical activities during the days on which they would be filmed and we found that, on average, less than 10% of the total time in the day was used for these activities. This result is consistent with the study by Strasser and collaborators (2009), who demonstrated that mathematics activities occupy 9% of educational time in preschool.

When making comparisons according to SES, we found that children in classrooms of high SES have approximately twice as many opportunities to participate in mathematical activities as children in low SES classrooms. In this study we observed that the high SES classrooms devote about 20 minutes of the day to teaching mathematics versus the low SES classrooms, which allocate approximately 10 minutes of the day to mathematical activities. This result is concerning for two reasons: first, because we see that in classrooms of all socioeconomic levels little time is devoted to teaching mathematics and, second, because it shows that children of low SES experience even less time on mathematical work than their peers in high SES, classrooms, because it is specifically these children who show lower competency in mathematical skills (Jordan et al., 1992; Jordan, Kaplan, Ramineni, & Locuniak, 2009) and, therefore, they need greater opportunities of involvement with early mathematical experiences.

With respect to the different types of content observed for teaching mathematics according to the SES of the classrooms in this study, we found that the classrooms with children of low SES emphasize experiences that mostly address content related to counting and cardinality, numerical decomposition, measurement and data, and pre-numerical activities; whereas the high SES classrooms focus on content related to basic operations, problem solving, definition of mathematical concepts, and geometry. The differences in the type of content on which work is done during teaching of mathematics—small in some cases, but significant in all the measured variables—may reflect the differences in the abilities that children had at the beginning of their educational process. Since children in high SES classrooms generally begin their educational process with greater skills than children of low SES (Jordan et al., 1992; 2009), it is possible that the emphasis on different content depending on the socioeconomic group responds to the different teaching needs of each group. In fact, according to the study by Jordan et al. (2009), the most disadvantaged children have different development curves to those of children from higher socioeconomic groups, since they have less experience in mathematics than the former, who obtain these experiences at home. According to the authors, these disadvantages of lower SES can be offset with more direct teaching of the basic knowledge of numbers and their composition. In this study the low SES classrooms emphasized more basic number skills than high SES classrooms, where skills were emphasized that included, in addition to knowledge of numbers, the use of numerical information in applied mathematical operations and explicit linking between the language and the mathematical concepts used.

If we consider this latter view to be true, our results show that, for all the levels of SES in our sample, the mathematical learning opportunities offered in the observed classrooms are highly homogeneous in terms of content, where practicing basic skills focused on numbers is predominant, such as counting and basic operations, rather than activities that emphasize mathematical comprehension, such as problem solving or the use of mathematical language. This perspective is problematic, because it shows that by the end of preschool, children in the observed classrooms receive learning experiences based on very elementary content for this level of education. In fact, the literature states that mathematical skills, like language skills, seem to have a sequential development trajectory, where the most basic skills (recognition of numbers, counting, and writing of numbers) are necessary for the acquisition of more advanced skills, such as operating with numbers or solving everyday problems (Galindo & Sonnenschein, 2015). In this regard, offering children experiences considered elementary at the end of preschool may indicate that children are developing these skills late and this is related to lower future mathematical performance. Therefore, it is important to consider that not only does it matter on what type of mathematical content work is done during preschool, but it is also important to address this mathematical content at the times when children are prepared to receive it.

Although the results of this study offer a first approach on what children are learning in mathematics during preschool education, this work is not without its limitations. Firstly, this study only considers 14 classrooms in which the children included in this study are grouped, so in no way is it a representative sample of what happens in other classrooms throughout the country.

Secondly, this research is transversal, which implies that no previous or subsequent measurements of the data from the participating classrooms were considered. Future studies should consider the incorporation of measurements at more points in time, so that they can learn about the teaching of mathematics at different times throughout the year and thus include a broader vision of the process of teaching mathematics at this educational level.

Finally, this research analyzes the characteristics of mathematics teaching at preschool, which take place infrequently in the classrooms observed according to the results of both this study and previous research (Strasser et al., 2009). This fact is in itself a limitation, since characterizing and analyzing mathematics teaching in such small spaces during the work day only offers us a limited vision of how children are learning mathematics at their educational centers.

Despite this, although this study has limitations, our results allow us to project future research regarding mathematical teaching during preschool education. In particular, this study raises questions about the quality of mathematics education at the preschool level in Chile and the opportunities for mathematical learning available to children of different socioeconomic groups.

Finally, strengthening the initial training of preschool educators in mathematics, improving the curriculum for preschool education, and incorporating enriched daily teaching practices in the classroom are all steps that we have to take if we are interested in providing learning opportunities in the early childhood classroom that allow children of all socioeconomic levels to develop skills for mathematical reasoning. Children learn and use mathematics every day, but without explicit support that allows them to transition from informal mathematical experiences to formal educational content it is likely that these children will not be able to develop their full potential and bad experiences in terms of the subject will be perpetuated.

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